Chapter 7: Similar Triangles and Trigonometry

Getting Started, page 370

1. a) ii) This is a relationship between two numbers, 3 and 5; so, it is a ratio.
b) iv) This is an equation relating two equivalent ratios; so, it is a proportion.
c) v) The two triangles have the same three side and angle measures; so, they are congruent triangles.
d) iii) A square is drawn on each side of the right triangle. Since the sum of the area of the blue square (a^2) and the area of the pink square (b^2) is equal to the area of the purple square (c^2), this diagram illustrates the Pythagorean theorem ($a^2 + b^2 = c^2$).
e) i) The highlighted side length is the longest side of the right triangle (or opposite the right angle), which is the hypotenuse.
f) vi) The highlighted angle is 30° and an acute angle is any angle less than 90°.

2. a)
$$\frac{x}{4} = \frac{9}{36}$$
$$4\left(\frac{x}{4}\right) = 4\left(\frac{9}{36}\right)$$
$$x = \frac{36}{36}$$
$$x = 1$$

b)
$$\frac{2}{5} = \frac{b}{20}$$
$$20\left(\frac{2}{5}\right) = 20\left(\frac{b}{20}\right)$$
$$\frac{40}{5} = b$$
$$8 = b$$

c)
$$\frac{2}{7} = \frac{5}{c}$$
$$c\left(\frac{2}{7}\right) = c\left(\frac{5}{c}\right)$$
$$\frac{2c}{7} = 5$$
$$7\left(\frac{2c}{7}\right) = 7(5)$$
$$2c = 35$$
$$c = 17.5$$

d)
$$\frac{2d}{9} = \frac{12}{4}$$
$$9\left(\frac{2d}{9}\right) = 9\left(\frac{12}{4}\right)$$
$$2d = \frac{108}{4}$$
$$2d = 27$$
$$d = 13.5$$

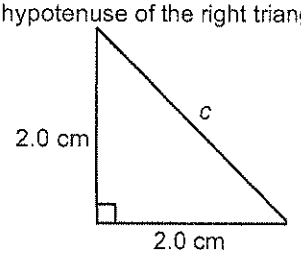
e)
$$\frac{a}{5.2} = \frac{7.8}{12.0}$$
$$5.2\left(\frac{a}{5.2}\right) = 5.2\left(\frac{7.8}{12.0}\right)$$
$$a = \frac{40.56}{12.0}$$
$$a = 3.38$$

f)
$$\frac{4.5}{y} = \frac{5.4}{2.4}$$
$$y\left(\frac{4.5}{y}\right) = y\left(\frac{5.4}{2.4}\right)$$
$$4.5 = \frac{5.4y}{2.4}$$
$$2.4(4.5) = 2.4\left(\frac{5.4y}{2.4}\right)$$
$$10.8 = 5.4y$$
$$2 = y$$

3. a) The unknown side length, c, is the hypotenuse.
$$c^2 = 3.6^2 + 4.8^2$$
$$c^2 = 12.96 + 23.04$$
$$c^2 = 36$$
$$c = \sqrt{36}$$
$$c = 6$$
The length of side c is 6.0 m.

b) c = 17.5 cm
$$17.5^2 = 14.0^2 + b^2$$
$$306.25 = 196 + b^2$$
$$306.25 - 196 = b^2$$
$$110.25 = b^2$$
$$\sqrt{110.25} = b$$
$$10.5 = b$$
The length of side b is 10.5 cm.

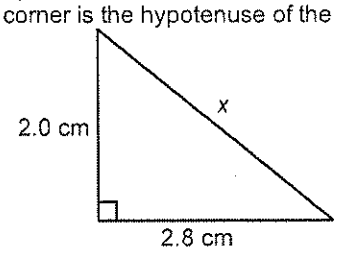
4. a) The side length of the sugar cube is 2.0 cm. The distance from corner to corner across the top is the hypotenuse of the right triangle shown.



$$c^2 = 2.0^2 + 2.0^2$$
$$c^2 = 4 + 4$$
$$c^2 = 8$$
$$c = \sqrt{8}$$
$$c \doteq 2.8$$

The distance from corner to corner across the top of the sugar cube is about 2.8 cm.

b) The distance from the top corner to the bottom corner is the hypotenuse of the right triangle.



$x^2 = 2.0^2 + 2.8^2$
$x^2 = 4 + 7.84$
$x^2 = 11.84$
$x = \sqrt{11.84}$
$x \doteq 3.4$

The distance from the top corner to the bottom corner of the sugar cube is about 3.4 cm, or 3.5 cm depending on rounding.

5. a) 10:14 = 5:7

b) $\frac{7.5}{15} = \frac{1}{2}$

c) –8:2 = –4:1

d) $\frac{27}{36} = \frac{3}{4}$

6. a) The interior angles in a triangle add to 180°.
$x = 180° - 107° - 42°$
$x = 31°$

b) The opposite angles produced by two intersecting lines are equal. Therefore, $x = 33°$.

c) The line intersects two parallel lines. Therefore, the alternate interior angles are equal; so, $x = 74°$.

d) The line intersects two parallel lines. The corresponding angles are equal and the supplementary interior angles add to 180°. Therefore:
$x = 180° - 120°$
$x = 60°$

7. a) Congruent. They are the same shape and size—the side lengths and angle measures are the same for both figures.

b) Similar. They are the same shape but different sizes—the angle measures are the same but the side lengths are different.

8. Two triangles are congruent if they have the same two angle measures and the same side measure between the two known angles. Therefore, I need the length of the side between the two 50° angles.

9. Set up a proportion and solve for x.

$$\frac{1}{1100} = \frac{3.7}{x}$$
$$x\left(\frac{1}{1100}\right) = x\left(\frac{3.7}{x}\right)$$
$$\frac{x}{1100} = 3.7$$
$$x = 4070$$

The building is 4070 cm or 40.7 m tall.

10. Answers may vary, e.g.,

a) You find equal angles when they are opposite angles. You find equal angles when they are corresponding angles in a case with parallel lines.

b) Angles add up to 180° when they are supplementary. Angles add up to 180° when they are the three interior angles of a triangle.

Lesson 7.1: Congruence and Similarity in Triangles, page 378

1. Check the angles.

$\angle C = 180° - 36° - 67°$
$= 77°$
$\angle A = \angle D$
$= 36°$
$\angle B = \angle E$
$= 67°$
$\angle C = \angle F$
$= 77°$

Check the sides.

$\frac{AB}{DE} = \frac{5.4}{1.8}$
$= 3$
$\frac{BC}{EF} = \frac{3.3}{1.1}$
$= 3$
$\frac{AC}{DF} = \frac{5.1}{1.7}$
$= 3$

The corresponding angles are equal and the corresponding sides are proportional.
Therefore, $\triangle ABC \sim \triangle DEF$.

2. a) A triangle that is congruent to $\triangle ABC$ will have the same side and angle measures. $\triangle DEF$ and $\triangle GHI$ are too small. $\triangle JKL$ is too large. Consider $\triangle MNO$:

$NO = 6$ cm
$= BC$
$\angle M = \angle A, \angle N = \angle B, \angle O = \angle C$

$\triangle ABC$ and $\triangle MNO$ have two angles equal and one side between the known angles equal.
Therefore, $\triangle ABC \cong \triangle MNO$.

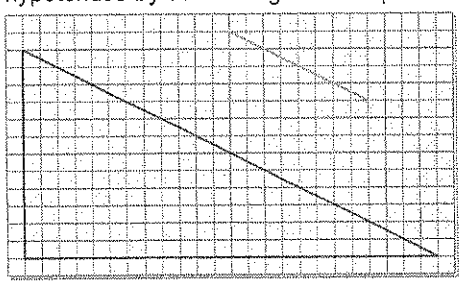
b) A triangle that is similar to $\triangle ABC$ will have proportional side measures and equal angle measures.

Consider $\triangle DEF$:
$\angle D = \angle B$
$\frac{AB}{DF} = \frac{8}{4}$
$= 2$
$\frac{BC}{DE} = \frac{6}{3}$
$= 2$
$\triangle ABC \sim \triangle DEF$

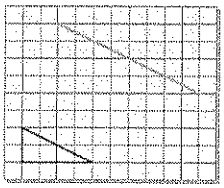
Consider $\triangle GHI$:
$\angle G = \angle B$
$\frac{AB}{GH} = \frac{8}{2}$
$= 4$
$\frac{BC}{GI} = \frac{6}{1.5}$
$= 4$
$\frac{AC}{HI} = \frac{10}{2.5}$
$= 4$
$\triangle ABC \sim \triangle GHI$

Consider $\triangle JKL$:
$\angle J = \angle A, \angle L = \angle B, \angle K = \angle C$
$\triangle ABC \sim \triangle JKL$

3. a) In the given triangle, the short vertical side length is 4 units and the long horizontal side length is 8 units. So, the enlargement should have side lengths that are 12 units and 24 units long. Draw the two side lengths and then draw the hypotenuse by connecting their endpoints.



b) In the given triangle, the short vertical side length is 4 units and the long horizontal side length is 8 units. The reduction should have side lengths that are 2 units by 4 units.



Draw the side lengths and then draw the hypotenuse by connecting their endpoints.

4. a) i) Three pairs of corresponding angles are equal and no corresponding sides are equal. Therefore, $\triangle ABC \sim \triangle DEF$.

ii) $\angle ABC = \angle DEF$
$\angle BCA = \angle EFD$
$\angle CAB = \angle FDE$

$\frac{DE}{AB} = \frac{4}{8}$ $\quad$ $\frac{EF}{BC} = \frac{2}{4}$
$= 0.5$ $\quad$ $= 0.5$

Scale factor is 0.5.

b) i) Three pairs of corresponding angles are equal and no corresponding sides are equal. Therefore, $\triangle MNO \sim \triangle OPQ$.

ii) $\angle MNO = \angle QPO$
$\angle NOM = \angle POQ$
$\angle OMN = \angle OQP$

$\frac{QP}{MN} = \frac{2.5}{5.0}$
$= 0.5$

The triangles share $\angle O$. Scale factor is 0.5.

c) i) Three pairs of corresponding angles are equal. Check the sides.

$$10^2 = 8^2 + GH^2$$
$$100 = 64 + GH^2$$
$$100 - 64 = GH^2$$
$$36 = GH^2$$
$$\sqrt{36} = GH$$
$$6 = GH$$

If $GH = 6$ cm, then $KJ = 10$ cm.
Three pairs of corresponding sides are equal.
Therefore, $\triangle GHI \cong \triangle JKL$.

ii) $GH = JL$
$HI = LK$
$IG = KJ$

$\angle GHI = \angle JLK$
$\angle HIG = \angle LKJ$
$\angle IGH = \angle KJL$

d) i) Three pairs of corresponding angles are equal and no corresponding sides are equal. Therefore, $\triangle RSV \sim \triangle TUV$.

ii) $\angle RSV = \angle UTV$
$\angle SVR = \angle TVU$
$\angle VRS = \angle VUT$

$\frac{UT}{RS} = \frac{4.2}{7.0}$
$= 0.6$

The triangles share $\angle V$. Scale factor is 0.6.

5. There are no angle measures shown, so check the proportions between corresponding sides.

$\frac{A'B'}{AB} = \frac{11.34}{5.68}$ $\quad$ $\frac{B'C'}{BC} = \frac{14.62}{7.31}$ $\quad$ $\frac{A'C'}{AC} = \frac{7.26}{3.63}$
$= 2$ $\quad$ $= 2$ $\quad$ $= 2$

The sides are in proportion. Therefore, $\triangle A'B'C' \sim \triangle ABC$.

6. a) $\angle L = 90°$. The corresponding vertices of similar triangles must be listed in the same order; therefore $\angle L$ corresponds to $\angle P$.

b) Set up a proportion between PQ and LM to calculate the scale factor.

$$\frac{PQ}{LM} = \frac{15}{5}$$
$$= 3$$

The scale factor is 3.

$PR = 3LN$ $\quad$ $QR = 3MN$
$= 3(12)$ $\quad$ $= 3(13)$
$= 36$ $\quad$ $= 39$

PR is 36 cm. $\quad$ QR is 39 cm.

7. a) Three pairs of corresponding angles and one pair of corresponding sides is equal; therefore $\triangle ABC \cong \triangle DEF$.

$\angle F = \angle y$
$= 30°$
$\angle D = 180° - 90° - 30°$
$= 60°$
$\angle D = \angle x$
$= 60°$

b) Three pairs of corresponding angles are equal and the sides are proportional; so, $\triangle KLM \sim \triangle NOP$. Set up a proportion to determine the scale factor.

$$\frac{NO}{KM} = \frac{6.0}{12.0}$$
$$= 0.5$$
$$b = 0.5(11.0)$$
$$= 5.5$$

Side length b is 5.5 cm.
Use the Pythagorean theorem to determine c.

$$12^2 = 11^2 + c^2$$
$$144 = 121 + c^2$$
$$144 - 121 = c^2$$
$$23 = c^2$$
$$\sqrt{23} = c$$
$$4.8 \doteq c$$

Side length c is about 4.8 cm.

c) Three pairs of corresponding angles and two pairs of corresponding sides are equal; therefore $\triangle GHI \cong \triangle IJG$.

$a = GJ$
$= 8$

Side length a is 8 m.

$\angle H = 180° - 140°$ (supplementary angles)
$= 40°$
$\angle H = \angle z$
$= 40°$

d) Three pairs of corresponding angles are equal and the sides are proportional but not equal; therefore $\triangle QRS \sim \triangle UTS$. Set up a proportion to determine the scale factor.

$$\frac{RS}{TS} = \frac{8.0}{4.0}$$
$$= 2$$
$$c = 2TU$$
$$= 7.0$$

The side length c is 7.0 cm.

Use the Pythagorean theorem to determine d.

$$d^2 = 8.0^2 + 7.0^2$$
$$d^2 = 64 + 49$$
$$d^2 = 113$$
$$d = \sqrt{113}$$
$$d \doteq 10.6$$
$$e = 0.5d$$
$$= 0.5(10.6)$$
$$\doteq 5.3$$

So, d is about 10.6 cm and e is about 5.3 cm.

8. a) Three pairs of corresponding angles and two pairs of corresponding sides are equal; therefore $\triangle ABC \cong \triangle EDC$.

$$b = BC$$
$$= 5$$

The side length b is 5 cm.

$\angle ACB = 40°$ (they're opposite angles)

$$\angle a = 180° - 80° - 40°$$
$$= 60°$$

b) Each pair of triangles has three equal side lengths (two marked side lengths and one shared side length); therefore, $\triangle ABC \cong \triangle ADC$, $\triangle ADE \cong \triangle ABE$, and $\triangle CDE \cong \triangle CBE$.

$$\angle d = \angle CBE$$
$$= 40°$$

$\angle c = 180° - 30° - 90°$ $\quad$ $\angle e = 180° - 40° - 90°$

$= 60°$ $\quad$ $= 50°$

c) Three pairs of corresponding angles are equal and the sides are proportional but not equal; therefore $\triangle ACE \sim \triangle BCD$. Set up a proportion to determine the scale factor.

$$\frac{CD}{CE} = \frac{4}{4+5}$$
$$= \frac{4}{9}$$
$$g = \frac{4}{9}AE$$
$$= \frac{4}{9}(9)$$
$$= 4$$

The side length g is 4 cm.

$AE = CE$ and $CD = BD$; therefore $\triangle ACE$ and $\triangle BCD$ are both isosceles triangles and $\angle f = \angle h$.

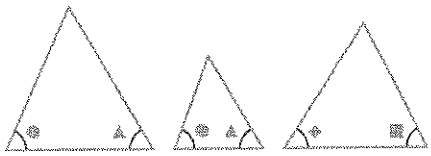
$$180° = 70° + 2\angle f$$
$$180° - 70° = 2\angle f$$
$$110° = 2\angle f$$
$$55° = \angle f$$
$$\angle f = 55°$$
$$\angle f = \angle h$$

d) Three pairs of corresponding angles are equal and the sides are proportional but not equal; therefore $\triangle ABC \sim \triangle EDC$. Set up a proportion to determine the scale factor.

$$\frac{CD}{BC} = \frac{2.0}{5.0}$$
$$= 0.4$$
$$i = 0.4AB$$
$$= 0.4(6.0)$$
$$= 2.4$$

The side length i is 2.4 cm.

9. Answers may vary, e.g.,



The first two triangles are similar because three pairs of corresponding angles are equal and the sides are proportional but not equal. The third triangle has different angle measures, so it is not similar to the first two.

10. Each angle in an equilateral triangle is always 60°, so any two equilateral triangles will always be similar. In a right triangle, the two non-right angles can be different, so right triangles will not always be similar. In an isosceles triangle, two angles are always equal but they can be different values, so isosceles triangles will not always be similar.

11. Three pairs of corresponding angles are equal and the sides are proportional but not equal; therefore, $\triangle ADE \sim \triangle ABC$. Use the Pythagorean theorem to determine BC.

$$\left(\sqrt{13}\right)^2 = 3^2 + BC^2$$
$$13 = 9 + BC^2$$
$$13 - 9 = BC^2$$
$$4 = BC^2$$
$$2.0 = BC$$

Side BC is 2.0 cm.

Set up a proportion to determine the scale factor.

$$\frac{DE}{BC} = \frac{2.5}{2.0} = 1.25$$

Use the scale factor to determine DA.

$$DA = 1.25BA$$
$$= 1.25(3)$$
$$= 3.75$$

Use DA and BA to determine DB.

$$DB = DA - BA$$
$$= 3.75 - 3$$
$$= 0.75$$

Side length DB is 0.75 cm.

12. The angles opposite each other are equal. Since the red triangle has two sides equal, it is isosceles. So, the base angles of this triangle are equal. The sum of the base angles in the red triangle equals the sum of the base angles in the black triangle. Since the bases of the triangles are parallel, the black triangle is isosceles. Three pairs of corresponding angles are equal and the sides are proportional but not equal; therefore the triangles are similar. Set up a proportion to determine the scale factor.

$$\frac{\text{base black}}{\text{base red}} = \frac{2.6}{1.2} \doteq 2.1667$$

Calculate the unknown side lengths on the black triangle.

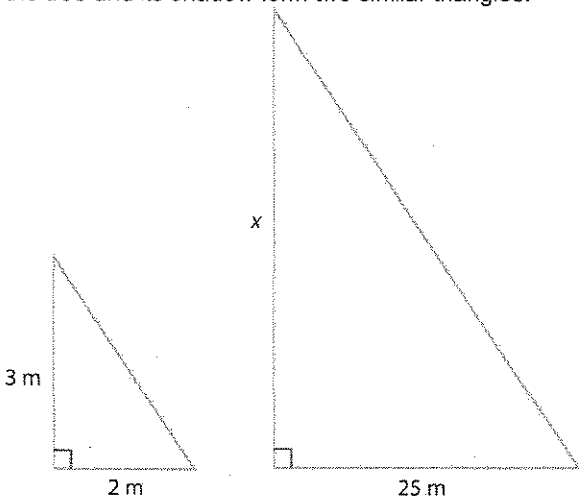
$1.5(2.1667) \doteq 3.25$

Calculate the total perimeter.

$1.5 + 1.2 + 1.5 + 3.25 + 2.6 + 3.25 = 13.3$

The perimeter of the logo is 13.3 cm.

13. Both the tree and the building make a right angle with the ground. The angle formed between the top of the tree and the end of its shadow and angle between the top of the building and the end of its shadow are the same because the sun is in the same position in each case. Therefore the building and its shadow and the tree and its shadow form two similar triangles.



Let the height of the building be x. Set up a proportion using the corresponding side lengths and solve for x.

$$\frac{25}{2} = \frac{x}{3}$$

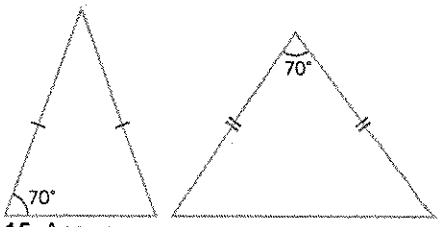
$$3\left(\frac{25}{2}\right) = 3\left(\frac{x}{3}\right)$$

$$\frac{75}{2} = x$$

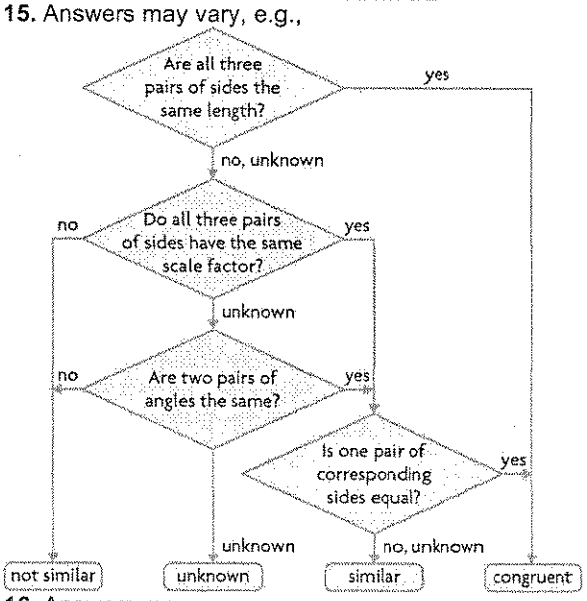
$$37.5 = x$$

The building is 37.5 m or about 38 m tall.

14. Not necessarily because it depends on where the angles are located in each triangle. If each equal angle is between the pair of equal side lengths in each triangle, then the triangles will be similar. In a situation like that shown below, the triangles would not be similar.



15. Answers may vary, e.g.,



16. Answers may vary, e.g.,

Show the triangles are similar.

$\angle BCA = \angle ACD = \angle BAD = 90°$

$\angle ABC = \angle DBA$; therefore, $\triangle BCA \sim \triangle BAD$.

$\angle CDA = \angle ADB$; therefore, $\triangle BCA \sim \triangle BAD$.

Find the ratios of the corresponding sides.

$$\frac{AB}{BD} = \frac{BC}{AB}$$

$$AB \times \frac{AB}{BD} = \frac{BC}{AB} \times AB \quad \text{Multiply both sides by } AB.$$

$$\frac{AB^2}{BD} = BC$$

$$BD \times \frac{AB^2}{BD} = BC \times BD \quad \text{Multiply both sides by } BD.$$

$$AB^2 = BC(BD)$$

$$\frac{AD}{BD} = \frac{CD}{AD}$$

$AD \times \frac{AD}{BD} = \frac{CD}{AD} \times AD$ Multiply both sides by AD.

$$\frac{AD^2}{BD} = CD$$

$BD \times \frac{AD^2}{BD} = CD \times BD$ Multiply both sides by BD.

$$AD^2 = CD(BD)$$

Use the ratios to prove the theorem.

$AB^2 + AD^2 = BC(BD) + CD(BD)$

$AB^2 + AD^2 = BD(BC + CD)$

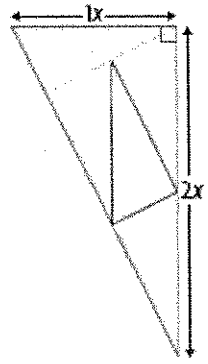
Since $BC + CD = BD$, substitute BD for $BC + CD$ into the equation.

$AB^2 + AD^2 = BD(BD)$

$AB^2 + AD^2 = BD^2$

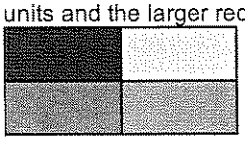
The Pythagorean theorem has been proven.

17. First, create two interior right triangles by drawing the altitude of the original triangle from the vertex of the 90° angle to the hypotenuse (yellow line). Next, mark the midpoints of the three sides on the larger interior triangle (vertices of the red triangle). Finally, join all the midpoints to divide the larger interior triangle into four small similar right triangles (red lines). They will be the same size as the first small interior right triangle making a total of five congruent triangles.



18. Answers may vary, e.g.,

I used four small similar rectangles to create a larger similar rectangle. Each small rectangle is 1 unit by 2 units and the larger rectangle is 2 units by 4 units.



Lesson 7.2: Solving Similar Triangle Problems, page 385

1. Answers may vary, e.g.,

a) $\angle ABC = \angle EBD$ (they're opposite angles)

The sum of the angles in a triangle is 180°, so the sum of $\angle A$ and $\angle C$ equals the sum of $\angle D$ and $\angle E$. $\angle A = \angle C$ because $\triangle ABC$ is isosceles, and $\angle D = \angle E$ because $\triangle EBD$ is isosceles. So, $\angle A = \angle C = \angle D = \angle E$. Therefore, the corresponding angles are equal.

b) Set up a proportion to calculate the scale factor. Answers may vary, e.g.,

$$\frac{AB}{BE} = \frac{8}{2}$$

$$= 4$$

The length of each side in $\triangle ACB$ is 4 times the length of each corresponding side in $\triangle EDB$.

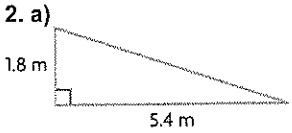
Or

$$\frac{BE}{AB} = \frac{2}{8}$$

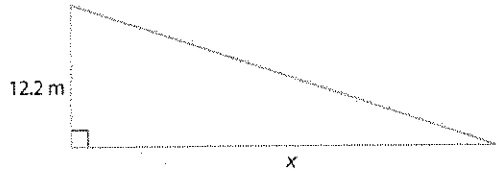
$$= 0.25$$

The length of each side in $\triangle EDB$ is 0.25 times the length of each corresponding side in $\triangle ACB$.

2. a)



b) Both Miki and the tree form right angles with the ground and the angle of elevation for the sun is the same in each case. Therefore, the triangle formed by the tree and its shadow will be similar to the triangle formed by Miki and his shadow. That means the ratio of the tree's height to Miki's height is equal to the ratio of their shadows.



Let x represent the length of the tree's shadow. Set up a proportion and solve for x.

$$\frac{12.2}{1.8} = \frac{x}{5.4}$$

$$5.4\left(\frac{12.2}{1.8}\right) = 5.4\left(\frac{x}{5.4}\right)$$

$$\frac{65.88}{1.8} = x$$

$$36.6 = x$$

The tree's shadow is 36.6 m long.

3. a) Both triangles share $\angle C$.

$\angle B = \angle E = 90°$

Two pairs of corresponding angles are equal; therefore, $\triangle ABC \sim \triangle DEC$.

b) Set up a proportion and solve for x.

$$\frac{x}{5} = \frac{8+9}{9}$$

$$5\left(\frac{x}{5}\right) = 5\left(\frac{17}{9}\right)$$

$$x = \frac{85}{9}$$

$$x \doteq 9$$

4. a) The hypotenuse represents the distance from the top of Brian's head to the end of his shadow.
b) Both Brian and the CN Tower form right angles with the ground. The angle of elevation of the sun is the same in both cases. Two corresponding angles are equal; therefore, the triangles are similar.
c) Set up a proportion to determine the scale factor. Answers may vary, e.g.,

$$\frac{1229.5}{4.0} \doteq 307.4$$

The length of each side in CN Tower triangle is about 307.4 times the length of each corresponding side in Brian's triangle.
Or

$$\frac{4.0}{1229.5} \doteq 0.0033$$

The length of each side in Brian's triangle is about 0.0033 times the length of each corresponding side in the CN Tower triangle.
d) Multiply Brian's height by the larger scale factor, 307.4.
$307.4(1.8) \doteq 553.3$
The CN Tower is about 553.3 m tall.
5. Two pairs of corresponding angles are equal ($\angle B = \angle D$ and $\angle C$ is shared); therefore, $\triangle ABC \sim \triangle EDC$. Set up a proportion and solve for x.

$$\frac{30.0}{18.0} = \frac{x}{15.0}$$

$$15.0\left(\frac{30.0}{18.0}\right) = 15.0\left(\frac{x}{15.0}\right)$$

$$\frac{450.0}{18.0} = x$$

$$25.0 = x$$

The bay is 25.0 m wide.
6. a) Answers may vary, e.g.,
$\angle B = \angle E = 90°$
$\angle BCA = \angle ECD$ (they're opposite angles)
Two pairs of corresponding angles are equal; therefore, $\triangle ABC \sim \triangle DEC$.
b) The corresponding known sides in each triangle, $\frac{BC}{EC}$.

c) Set up a proportion and solve for AB.

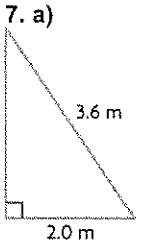
$$\frac{740}{37} = \frac{AB}{26}$$

$$26\left(\frac{740}{37}\right) = 26\left(\frac{AB}{26}\right)$$

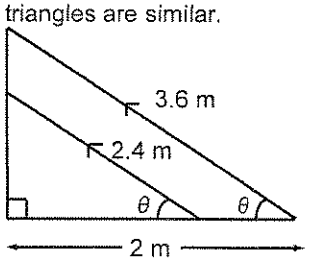
$$\frac{19\,240}{37} = AB$$

$$520 = AB$$

The distance across the gorge is 520 m.
7. a)



b) Answers may vary, e.g., The wall forms a right angle with the ground in both cases. The ladders are parallel; therefore the angles formed by the ladders and the ground are equal. Two pairs of corresponding angles are equal; therefore the triangles are similar.



c) Use the Pythagorean theorem to determine the distance, d, the tallest ladder reaches up the wall,

$$3.6^2 = d^2 + 2.0^2$$

$$12.96 - 4 = d^2$$

$$8.96 = d^2$$

$$\sqrt{8.96} = d$$

$$3.0 \doteq d$$

Length d is about 3.0 m.
Use a proportion to determine the distance, s, the shorter ladder reaches up the wall.

$$\frac{3.0}{3.6} = \frac{s}{2.4}$$

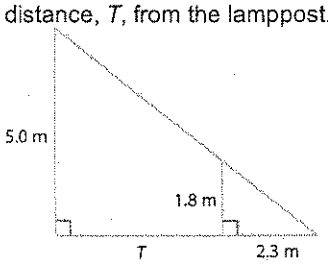
$$2.4\left(\frac{3.0}{3.6}\right) = 2.4\left(\frac{s}{2.4}\right)$$

$$\frac{7.2}{3.6} = s$$

$$2.0 = s$$

The 3.6-m ladder reaches about 3.0 m up the wall and the 2.4-m ladder reaches about 2.0 m up the wall.

8. Tyler and the lamppost both form a right angle with the ground. Since Tyler is parallel to the lamppost, the angle from the top of his head and from the top of the lamppost to the end of his shadow is the same. Since two corresponding angles are equal, the triangles formed by the lamppost, Tyler, and his shadow are similar. Set up a proportion to solve for Tyler's distance, T, from the lamppost.



$$\frac{2.3}{1.8} = \frac{2.3+T}{5.0}$$
$$5.0\left(\frac{2.3}{1.8}\right) = 5.0\left(\frac{2.3+T}{5.0}\right)$$
$$\frac{11.52}{1.8} = 2.3+T$$
$$6.4-2.3 \doteq T$$
$$4.1 \doteq T$$

Tyler is about 4.1 m from the lamppost.

9. Two pairs of corresponding angles are equal; therefore the two triangles formed are similar. Set up a proportion and solve for the height of the telephone pole, T

$$\frac{3.00}{5.00} = \frac{T-7.46}{T}$$
$$T\left(\frac{3.00}{5.00}\right) = T\left(\frac{T-7.46}{T}\right)$$
$$0.60T = T-7.46$$
$$7.46 = T-0.60T$$
$$7.46 = 0.40T$$
$$\frac{7.46}{0.40} = T$$
$$18.65 \doteq T$$

The telephone pole is 18.65 m tall.

10. There is not enough information given to determine if the sales clerk is correct or not. Need to know the proportions of the screens.

11. All three triangles have one right angle and share one angle. Since two corresponding angles are equal, the three triangles are similar. Set up a proportion for determine the unknown distance, d, from the 1.0-m pole to the end of the triangle.

$$\frac{1.6}{1.0} = \frac{12.0+d}{d}$$
$$d(1.6) = d\left(\frac{12.0+d}{d}\right)$$
$$1.6d = 12.0+d$$
$$1.6d-d = 12.0$$
$$0.6d = 12.0$$
$$d = 20$$

The distance is 20 m.

Set up a proportion to determine the height of the hill, h.

$$\frac{1.0}{20.0} = \frac{h}{20.0+12.0+415.0}$$
$$447.0\left(\frac{1.0}{20.0}\right) = 447.0\left(\frac{h}{447.0}\right)$$
$$\frac{447.0}{20.0} = h$$
$$22.35 = h$$

The hill is 22.35 m tall.

12. Two pairs of corresponding angles are equal; therefore the two triangles formed are similar. Set up a proportion and solve for the height of the window, w.

$$\frac{5.0}{0.5} = \frac{w}{1.6}$$
$$1.6\left(\frac{5.0}{0.5}\right) = 1.6\left(\frac{w}{1.6}\right)$$
$$\frac{8.0}{0.5} = w$$
$$16.0 = w$$

The window is 16.0 m high.

13. a) I need to know one of the two other unknown sides: AC or EF.

b) If I knew AC, I could use the proportion $\frac{AC}{DF} = \frac{AB}{x}$ to solve for x and the Pythagorean theorem to solve for y. If I knew EF, I could use the Pythagorean theorem to solve for x and then the proportion $\frac{y}{EF} = \frac{AB}{x}$ to solve for y.

14. Two pairs of corresponding angles are equal; therefore $\triangle ABC \sim \triangle EDC$. Use the Pythagorean theorem to solve for BC.

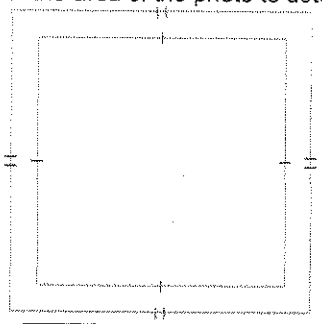
$$204^2 = 96^2 + BC^2$$
$$41\,616 = 9216 + BC^2$$
$$41\,616 - 9216 = BC^2$$
$$32\,400 = BC^2$$
$$\sqrt{32\,400} = BC$$
$$180 = BC$$

Set up a proportion and solve for DE.

$$\frac{96}{180} = \frac{DE}{396-180}$$
$$\frac{96}{180} = \frac{DE}{216}$$
$$216\left(\frac{96}{180}\right) = 216\left(\frac{DE}{216}\right)$$
$$\frac{20\,736}{180} = DE$$
$$115 \doteq DE$$

The width of the river is about 115 m.

15. a) The photo is a square, so take the square root of the area of the photo to determine the side length.



$\sqrt{324.00} = 18$, so photo side length is 18.00 cm.
To determine the side length of the frame, take the square root of the sum of the area of the frame and photo.
$\sqrt{324.00+142.56} \doteq 21.83$, so the frame side length is 21.83 cm.
Set up a proportion to determine the scale factor:

$$\frac{21.60}{18.00} = 1.2$$

The scale factor that relates the dimensions of the photo and the frame is 1.2.
b) The width of the frame is the total width less the width of the photo divided by two, since the frame is on both sides of the photo.

$$\frac{21.60 - 18.00}{2} = 1.8$$

The frame is 1.8 cm wide.

Mid-Chapter Review, page 390

1. a) By measuring, I know $\angle ABC = \angle EFD$ and $\angle CAB = \angle DEF$. By counting grid squares, I know $EF = AB$. If two corresponding angles and one pair of corresponding sides are equal, then $\triangle EFD \cong \triangle ABC$.
b) By measuring, I know $\angle ABC = \angle MON$ and $\angle CAB = \angle MNO$. By counting grid squares, I know $ON = 0.5AB$. Two corresponding angles are equal and one pair of corresponding side are proportional by a scale factor of 0.5, so $\triangle MON \sim \triangle ABC$.
2. a) $\angle ABC$ is angle B, the angle between AB and BC, so this corresponds to angle S, the angle between RS and ST. Therefore, $\angle ABC = \angle RST$.
b) $\angle BCA$ is angle C, the angle between BC and CA, so this corresponds to angle T, the angle between ST and TR. Therefore, $\angle BCA = \angle STR$.
c) AB and RS are corresponding sides, so this ratio will equal the ratio of any other two corresponding sides with a side from $\triangle ABC$ on top and a side from $\triangle RST$ on the bottom. Therefore,

$$\frac{AB}{RS} = \frac{BC}{ST} \quad \text{or} \quad \frac{AB}{RS} = \frac{CA}{TR}.$$

d) Vertex S corresponds with vertex B, vertex T corresponds with vertex C, and vertex R corresponds with vertex A. Therefore, $\triangle STR \sim \triangle BCA$.
e) ST and BC are corresponding sides, so this ratio will equal the ratio of any other two corresponding sides with a side from $\triangle RST$ on top and a side from $\triangle ABC$ on the bottom. Therefore, $\frac{ST}{BC} = \frac{TR}{CA}$ or $\frac{ST}{BC} = \frac{RS}{AB}$.
f) $\angle SRT$ is angle R, the angle between SR and RT, so this corresponds to angle A, the angle between BA and AC. Therefore, $\angle SRT = \angle BAC$.
3. Since $\triangle abc$ and $\triangle def$ have two equal corresponding angles, they are similar triangles. Since a is opposite $\angle o$ in $\triangle abc$ and f is opposite $\angle o$ in $\triangle fde$, then a and f are corresponding sides. Since c and e are both opposite $\angle x$ in their triangles, they are corresponding sides. This means that b and d are also corresponding sides. Therefore,

$$\frac{a}{f} = \frac{b}{d} = \frac{c}{e}.$$

4. True. If you know two of the angles, you can always calculate the third angle. If you know all of the angles in the two triangles, you can determine whether the triangles are similar.
5. Since the triangles have two equal corresponding angles, then they are similar and $\frac{4.1}{5.3} = \frac{y}{6.8} = \frac{x}{7.0}$.
Use the proportion to determine x and y.
Solving for x:

$$\frac{4.1}{5.3} = \frac{x}{7.0}$$
$$7.0\left(\frac{4.1}{5.3}\right) = 7.0\left(\frac{x}{7.0}\right)$$
$$7.0\left(\frac{4.1}{5.3}\right) = x$$
$$x \doteq 5.4$$

Solving for y:

$$\frac{4.1}{5.3} = \frac{y}{6.8}$$
$$6.8\left(\frac{4.1}{5.3}\right) = 6.8\left(\frac{y}{6.8}\right)$$
$$6.8\left(\frac{4.1}{5.3}\right) = y$$
$$y \doteq 5.3$$

Side lengths $x \doteq 5.4$ units and $y \doteq 5.3$ units.
6. a) Since $\triangle ABC \sim \triangle DEF$,

$$\frac{AC}{DF} = \frac{BC}{EF}$$
$$\frac{4}{12} = \frac{BC}{15}$$
$$15\left(\frac{4}{12}\right) = 15\left(\frac{BC}{15}\right)$$
$$15\left(\frac{4}{12}\right) = BC$$
$$BC = 5$$

Side length BC is 5 units.

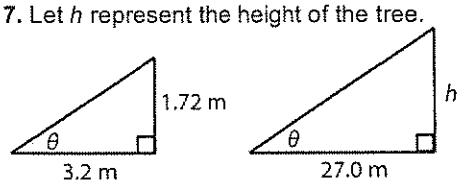
b) Since $\triangle ABC \sim \triangle DEF$,

$$\frac{DF}{AC} = \frac{DE}{AB}$$
$$\frac{12}{4} = \frac{DE}{6}$$
$$DE = 18$$

Side length DE is 18 units.

c) Check if the ratio of corresponding sides is equivalent. $\frac{IH}{FE} = \frac{6}{15}$ and $\frac{GH}{DE} = \frac{8}{18}$. Since $\frac{6}{15} \neq \frac{8}{18}$, the ratios are not equivalent and the triangles are not similar.

7. Let h represent the height of the tree.



Since Nora and the tree have the same angle of elevation, θ, and they are both perpendicular to the ground, the triangles formed by each have two equal corresponding angles so they are similar and

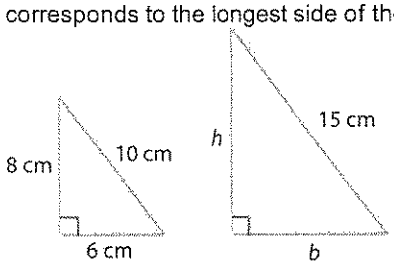
$$\frac{h}{1.72} = \frac{27}{3.2}.$$

Solve the proportion to determine h.

$$\frac{h}{1.72} = \frac{27}{3.2}$$
$$1.72\left(\frac{h}{1.72}\right) = 1.72\left(\frac{27}{3.2}\right)$$
$$h = 1.72\left(\frac{27}{3.2}\right)$$
$$h \doteq 14.5$$

So, the tree is about 14.5 m tall.

8. The longest side of the first triangle is 10 cm and corresponds to the longest side of the larger triangle.



Let b represent the base of the larger triangle, h represent the height of the larger triangle, and assume that the height is longer than the base. Write a proportion using the ratios of corresponding sides:

$$\frac{15}{10} = \frac{b}{6} = \frac{h}{8}$$

Use the proportion to determine b and h:

Solving for b:

$$\frac{15}{10} = \frac{b}{6}$$
$$6\left(\frac{15}{10}\right) = b$$
$$b = 9$$

Solving for h:

$$\frac{15}{10} = \frac{h}{8}$$
$$8\left(\frac{15}{10}\right) = h$$
$$h = 12$$

Find the perimeter:

The larger triangle has a base of 9 cm, a height of 12 cm, and a hypotenuse of 15 cm.

$$P = 9 + 12 + 15$$
$$= 36$$

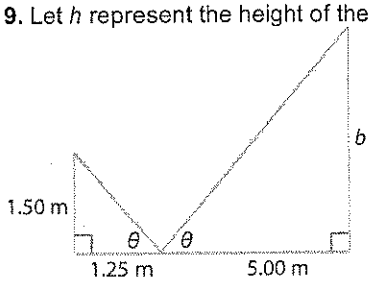
The perimeter is 36 cm.

Find the area:

$$A = \frac{b \times h}{2}$$
$$= \frac{9 \times 12}{2}$$
$$= 54$$

The area is 54 cm^2.

9. Let h represent the height of the flag pole.



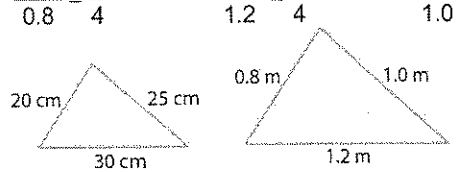
The angle of reflection, θ, is the same in both triangles formed and the flag pole and Connie are both perpendicular to the ground. Since both triangles have two equal corresponding angles, the triangles are similar. Write a proportion using the ratios of corresponding sides and then solve for h.

$$\frac{h}{1.50} = \frac{5.00}{1.25}$$
$$h = 1.50\left(\frac{5.00}{1.25}\right)$$
$$h = 6.00$$

The flagpole is 6.00 m tall.

10. Check the ratios of corresponding sides:

$$\frac{0.20}{0.8} = \frac{1}{4} \qquad \frac{0.30}{1.2} = \frac{1}{4} \qquad \frac{0.25}{1.0} = \frac{1}{4}$$



Since the ratios of corresponding sides are all equal, the triangles are similar, so, yes, Cam created his scale diagram correctly.

Lesson 7.3: Exploring Similar Right Triangles, page 393

1. a) From the diagram of $\triangle ABC$, opposite side is BC, the adjacent side is AB, and the hypotenuse is AC. In $\triangle DEF$, the opposite side is EF, the adjacent side is DE and the hypotenuse is DF.

b) i) $\frac{BC}{AC} = \frac{3}{5}$ and $\frac{EF}{FD} = \frac{12}{20} = \frac{3}{5}$.

ii) $\frac{AB}{AC} = \frac{4}{5}$ and $\frac{DE}{DF} = \frac{16}{20} = \frac{4}{5}$.

iii) $\frac{BC}{AB} = \frac{3}{4}$ and $\frac{EF}{DE} = \frac{12}{16} = \frac{3}{4}$.

2. a) From the diagram, $\triangle KJC$, $\triangle IHC$, $\triangle GFC$, and $\triangle EDC$ are all similar to $\triangle ABC$ because they each have a corresponding angle at C and a corresponding right angle.

b) i) From the diagram, the ratios are:

$\frac{AB}{AC}, \frac{KJ}{KC}, \frac{IH}{IC}, \frac{GF}{GC}$, and $\frac{ED}{EC}$.

ii) From the diagram, the ratios are:

$\frac{BC}{AC}, \frac{JC}{KC}, \frac{HC}{IC}, \frac{FC}{GC}$, and $\frac{DC}{EC}$.

iii) From the diagram, the ratios are:

$\frac{AB}{BC}, \frac{KJ}{JC}, \frac{IH}{HC}, \frac{GF}{FC}$, and $\frac{ED}{DC}$.

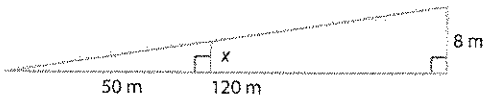
c) Since the triangles above are similar:

i) $\frac{AB}{AC} = \frac{KJ}{KC} = \frac{IH}{IC} = \frac{GF}{GC} = \frac{ED}{EC}$

ii) $\frac{BC}{AC} = \frac{JC}{KC} = \frac{HC}{IC} = \frac{FC}{GC} = \frac{DC}{EC}$

iii) $\frac{AB}{BC} = \frac{KJ}{JC} = \frac{IH}{HC} = \frac{GF}{FC} = \frac{ED}{DC}$

3. a) Let x represent the rise and write a proportion using equivalent $\frac{\text{rise}}{\text{run}}$ ratios. Then solve for x.

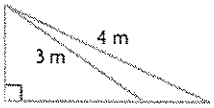


$$\frac{8}{120} = \frac{x}{50}$$
$$50\left(\frac{8}{120}\right) = x$$
$$x \doteq 3.3$$

The road will rise about 3 m.

b) $\frac{8}{20} = \frac{3.3}{50} = 0.67$; The slopes are the same because the rise and the run are from similar triangles.

4.



From the diagram, the 3 m ramp will have a greater angle of elevation.

Lesson 7.4: The Primary Trigonometric Ratios, page 398

1. a) From the diagram, BC is opposite to $\angle A$.

b) From the diagram, AC is adjacent to $\angle A$.

c) The hypotenuse is the side opposite the right angle, so from the diagram, AB is the hypotenuse.

2. Use the calculator to solve each ratio.

a) $\tan 34^\circ \doteq 0.6745$

b) $\sin 78^\circ \doteq 0.9781$

c) $\cos 49^\circ \doteq 0.6561$

d) $\sin 12^\circ \doteq 0.2079$

3. Use the calculator to solve each ratio.

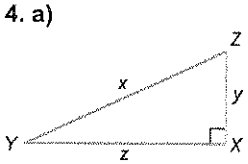
a) $\sin\theta = 0.5$
$\theta = \sin^{-1}(0.5)$
$\theta = 30^\circ$

b) $\tan\theta = 1$
$\theta = \tan^{-1}(1)$
$\theta = 45^\circ$

c) $\cos\theta = 0.5$
$\theta = \cos^{-1}(0.5)$
$\theta = 60^\circ$

d) $\sin\theta = 0.8660$
$\theta = \sin^{-1}(0.8660)$
$\theta \doteq 60^\circ$

4. a)



b) From the diagram,

$\sin Y = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{x}$ $\quad$ $\cos Y = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{z}{x}$

$\tan Y = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{z}$

5. a) $\sin C = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{17} \doteq 0.4706$

b) $\cos C = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{17} \doteq 0.8824$

c) $\tan B = \frac{\text{opposite}}{\text{adjacent}} = \frac{15}{8} = 1.8750$

d) $\tan C = \dfrac{\text{opposite}}{\text{adjacent}}$

$= \dfrac{8}{15}$

$\doteq 0.5333$

e) $\cos B = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$= \dfrac{8}{17}$

$\doteq 0.4706$

f) $\sin B = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$= \dfrac{15}{17}$

$\doteq 0.8824$

6. a) This statement is false.

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$= \dfrac{2.5}{5.6}$

$\doteq 0.4464$

$\neq 0.4$

b) This statement is true.

$\tan\alpha = \dfrac{\text{opposite}}{\text{adjacent}}$

$= \dfrac{5.0}{2.5}$

$= 2$

c) This statement is false.

$\cos\alpha = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$= \dfrac{2.5}{5.6}$

$\doteq 0.4464$

$\neq 0.8929$

d) This statement is true.

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$= \dfrac{5.0}{5.6}$

$\doteq 0.8929$

7. a) Using the sine ratio:

$\sin x = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$= \dfrac{60}{65}$

$x = \sin^{-1}\left(\dfrac{60}{65}\right)$

$x \doteq 67°$

x is about 67°.

b) No. After x is calculated, you can determine y using the fact that the sum of the angles in a triangle is 180°.

8. a) $\cos 45° = \dfrac{x}{6}$

$6\cos 45° = x$

$x \doteq 4.2$

Side length x is about 4.2 units.

b) $\sin 62° = \dfrac{x}{14}$

$14\sin 62° = x$

$x \doteq 12.4$

Side length x is about 12.4 units.

c) $\tan 75° = \dfrac{x}{20}$

$20\tan 75° = x$

$x \doteq 74.6$

Side length x is about 74.6 units.

d) $\tan 80° = \dfrac{12}{x}$

$x = \dfrac{12}{\tan 80°}$

$x \doteq 2.1$

Side length x is about 2.1 units.

e) $\cos 60° = \dfrac{15}{x}$

$x = \dfrac{15}{\cos 60°}$

$x = 30.0$

Side length x is about 30.0 units.

f) $\sin 45° = \dfrac{10}{x}$

$x = \dfrac{10}{\sin 45°}$

$x \doteq 14.1$

Side length x is about 14.1 units.

9. a) 50 is the length of the side adjacent to θ and 54 is the length of the hypotenuse, so $\dfrac{\text{adjacent}}{\text{hypotenuse}} = \cos\theta$.

b) 20 is the length of the side opposite to θ and 50 is the length of the side adjacent to θ, so $\dfrac{\text{opposite}}{\text{adjacent}} = \tan\theta$.

c) 20 is the length of the side opposite to θ and 54 is the length of the hypotenuse; therefore $\dfrac{\text{opposite}}{\text{hypotenuse}} = \sin\theta$.

10. a) Using the Pythagorean theorem:

$x^2 = 8^2 + 15^2$

$x = \sqrt{64 + 225}$

$x = 17$

Side length x is 17 cm.

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{15}{17}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{8}{17}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{15}{8}$

b) Using the Pythagorean theorem:

$x^2 + 12^2 = 13^2$

$x = \sqrt{169 - 144}$

$x = 5$

Side length x is 5 cm.

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{13}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{13}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{5}{12}$

11. a) $\tan\theta = \frac{15}{8}$

$\theta = \tan^{-1}\left(\frac{15}{8}\right)$

$\theta \doteq 61.9°$

b) $\cos\theta = \frac{12}{13}$

$\theta = \cos^{-1}\left(\frac{12}{13}\right)$

$\theta \doteq 22.6°$

12. a) $\sin\theta = \frac{2}{5}$

$\theta = \sin^{-1}\left(\frac{2}{5}\right)$

$\theta \doteq 23.6°$

b) $\cos\theta = \frac{4}{9}$

$\theta = \cos^{-1}\left(\frac{4}{9}\right)$

$\theta \doteq 63.6°$

c) $\tan\theta = 3$

$\theta = \tan^{-1}(3)$

$\theta \doteq 71.6°$

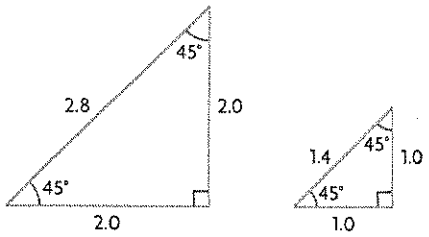
d) $\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta = 30°$

13. Not necessarily, it only means the ratio of the lengths of these two sides is 1:2. So, for instance, the adjacent side could be 20 cm and the hypotenuse could be 40 cm.

14. Answers may vary, e.g., I knew that, if tan was 1, then the adjacent and opposite sides were equal, so I drew two right isosceles triangles. To determine the hypotenuse for each, I used the Pythagorean theorem. To figure out the angles, I knew they were 45° because they are equal (isosceles triangles) and the sum of the angles in each was 180° (45° + 45° + 90° = 180°).



Since two sides in each triangle are the same measure, both triangles are isosceles. Also, since the angles in both triangles are 45°, 45°, and 90°, the triangles are similar.

15. a) Yes, if the selected angle is $\angle C$.

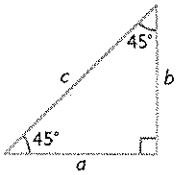
b) Yes, if the selected angle is $\angle A$.

c) No. The hypotenuse is always opposite the right angle.

16. If we have a triangle with sides, a, b, and c, where a is the side opposite to θ, b is the side adjacent to θ, and c is the hypotenuse, then, for sin θ to be equal to cos θ, $\frac{a}{c}$ must be equal to $\frac{b}{c}$. For $\frac{a}{c}$ to be equal to $\frac{b}{c}$, $a = b$ and we have an isosceles triangle.

Since the triangle is a right isosceles triangle and the sum of the interior angles is 180°, $\theta = \frac{180° - 90°}{2} = 45°$.

So, sin θ = cos θ when $\theta = 45°$.



17. Answers may vary, e.g., if θ increases but the side adjacent to it stays constant, then the side opposite to it must get longer. This is the reason why tan θ (the ratio of the opposite to the adjacent side) increases.

Lesson 7.5 Solving Right Triangles, page 403

1. a) $\cos 60° = \frac{x}{4.3}$

$4.3(\cos 60°) = x$

$x \doteq 2.2$

x is about 2.2 km.

b) $\tan 54° = \frac{x}{25.0}$

$25.0(\tan 54°) = x$

$x = 34.4$

x is 34.4 cm.

2. a) $\sin\theta = \frac{3}{5}$

$\theta = \sin^{-1}\left(\frac{3}{5}\right)$

$\theta \doteq 37°$

b) $\cos\theta = \frac{5}{8}$

$\theta = \cos^{-1}\left(\frac{5}{8}\right)$

$\theta \doteq 51°$

3. Use the fact that the interior angles of a triangle add to 180° to determine θ:

$$\theta + 90^\circ + 37^\circ = 180^\circ$$
$$\theta = 53^\circ$$

Solve the cosine ratio to determine a:

$$\cos 37^\circ = \frac{a}{18}$$
$$18(\cos 37^\circ) = a$$
$$a \doteq 14$$

Solve the sine ratio to determine b:

$$\sin 37^\circ = \frac{b}{18}$$
$$18 \sin 37^\circ = b$$
$$b \doteq 11$$

θ is 53°, a is about 14 cm, and b is about 11 cm.

4. a) $\sin A = 0.9063$

$$\angle A = \sin^{-1}(0.9063)$$
$$\angle A \doteq 65^\circ$$

b) $\cos A = \frac{4}{5}$

$$\angle A = \cos^{-1}\left(\frac{4}{5}\right)$$
$$\angle A \doteq 37^\circ$$

5. a) i) We know the values of two angles and the length of the hypotenuse. Side x is adjacent to the 40° angle and opposite to the 50° angle.

So, $\sin 50^\circ = \frac{x}{90}$ or $\cos 40^\circ = \frac{x}{90}$ could be used to determine x.

ii) Using the sine ratio:

$$\sin 50^\circ = \frac{x}{90}$$
$$90(\sin 50^\circ) = x$$
$$x \doteq 69$$

Side x is about 69 mm.

b) i) We know the values of two angles and the length of one side. The x is adjacent to the 39° angle and opposite the 51° angle.

So, $\tan 39^\circ = \frac{30}{x}$ or $\tan 51^\circ = \frac{x}{30}$ could be used to determine x.

ii) Using the tangent ratio:

$$\tan 51^\circ = \frac{x}{30}$$
$$30(\tan 51^\circ) = x$$
$$x \doteq 37$$

Side x is about 37 cm.

6. a) $\tan\theta = \frac{c}{a}$

$$\tan\theta = \frac{10}{10}$$
$$\theta = \tan^{-1}\left(\frac{10}{10}\right)$$
$$\theta = 45^\circ$$

b) $\sin\theta = \frac{c}{b}$

$$\sin\theta = \frac{6}{12}$$
$$\theta = \sin^{-1}\left(\frac{6}{12}\right)$$
$$\theta = 30^\circ$$

c) $\cos\theta = \frac{a}{b}$

$$\cos\theta = \frac{9}{15}$$
$$\theta = \cos^{-1}\left(\frac{9}{15}\right)$$
$$\theta \doteq 53^\circ$$

7. a) $\cos\angle A = \frac{20}{32}$

$$\angle A = \cos^{-1}\left(\frac{20}{32}\right)$$
$$\angle A \doteq 51^\circ$$

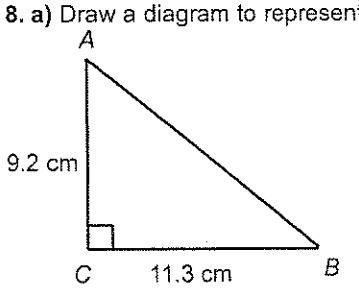
$$\sin\angle B = \frac{20}{32}$$
$$\angle B = \sin^{-1}\left(\frac{20}{32}\right)$$
$$\angle B \doteq 39^\circ$$

b) $\sin\angle A = \frac{16}{37}$

$$\angle A = \sin^{-1}\left(\frac{16}{37}\right)$$
$$\angle A \doteq 26^\circ$$

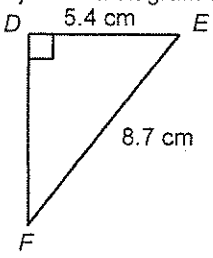
$$\cos\angle B = \frac{16}{37}$$
$$\angle B = \cos^{-1}\left(\frac{16}{37}\right)$$
$$\angle B \doteq 64^\circ$$

8. a) Draw a diagram to represent the situation.



$$\tan A = \frac{a}{b}$$
$$\tan A = \frac{11.3}{9.2}$$
$$\angle A = \tan^{-1}\left(\frac{11.3}{9.2}\right)$$
$$\angle A \doteq 51^\circ$$

b) Draw a diagram to represent the situation.



$$\sin F = \frac{f}{d}$$
$$\sin F = \frac{5.4}{8.7}$$
$$\angle F = \sin^{-1}\left(\frac{5.4}{8.7}\right)$$
$$\angle F = 38^\circ$$

9. Let h represent the height of the cliff.

$$\tan 70^\circ = \frac{h}{50}$$
$$50\tan 70^\circ = h$$
$$h \doteq 137$$

The cliff is about 137 m tall.

10. Since j is a perpendicular bisector, it bisects the base into two equal halves of length 4 cm each.
Use the right half of the triangle, to determine j:

$$\tan 60^\circ = \frac{j}{4}$$
$$4\tan 60^\circ = j$$
$$j \doteq 7$$

Side j is about 7 cm.
Use the left half of the triangle, to determine i:
Since the original triangle is isosceles, the angle adjacent to i is 60°.

$$\cos 60^\circ = \frac{4}{i}$$
$$i = \frac{4}{\cos 60^\circ}$$
$$i = 8$$

Side i is 8 cm.

11. The length of the ladder is the hypotenuse of the triangle created. Let l represent the length of the ladder,

$$\sin 75^\circ = \frac{2.8}{l}$$
$$l = \frac{2.8}{\sin 75^\circ}$$
$$l \doteq 2.9$$

The ladder is 2.9 m long.

12. a) This is incorrect. The side opposite A is 22.5, not 12, so her notes should read $\sin A = \frac{22.5}{25.5}$.

b) This is correct. The sum of the interior angles of the triangle must be 180°.

$$\angle A + \angle B + \angle C = 62^\circ + 28^\circ + 90^\circ$$
$$= 180^\circ$$

c) This is incorrect. The side adjacent to C is 22.5, not 25.5, and the hypotenuse is 25.5, not 22.5. Her notes should read $\cos C = \frac{22.5}{25.5}$.

d) This is correct. $\tan A = \frac{22.5}{12.0} = 1.875$

e) This is correct. $\sin C = \frac{24}{51} = \frac{12.0}{25.5}$

f) This is correct. $\tan C = \frac{12.0}{22.5} \doteq 0.53$

13. a) Using the Pythagorean theorem:

$$AC^2 = AB^2 + BC^2$$
$$AC^2 = 8^2 + 5^2$$
$$AC = \sqrt{8^2 + 5^2}$$
$$AC = \sqrt{64 + 25}$$
$$AC \quad 9$$

Side AC is 9 mm.
Use trigonometric ratios to determine the missing angles.

$$\tan A = \frac{5}{8}$$
$$\angle A = \tan^{-1}\left(\frac{5}{8}\right)$$
$$\angle A \doteq 32^\circ$$
$$\tan C = \frac{8}{5}$$
$$\angle C = \tan^{-1}\left(\frac{8}{5}\right)$$
$$\angle C \doteq 58^\circ$$

b) Use the fact that the angles of a triangle sum to 180° to determine $\angle L$:

$$\angle L = 180^\circ - 90^\circ - 72^\circ$$
$$\angle L = 18^\circ$$

Use trigonometric ratios to determine the missing sides.

$$\sin J = \frac{j}{10}$$
$$10\sin J = j$$
$$10\sin 72^\circ = j$$
$$10 \doteq j$$

Side length j is about 10 cm.

$$\sin L = \frac{l}{10}$$
$$10\sin L = l$$
$$10\sin 18^\circ = l$$
$$3 \doteq l$$

Side length l is about 3 cm.

c) Use the fact that the angles of a triangle sum to 180° to determine $\angle Q$:

$$\angle Q = 180^\circ - 90^\circ - 42^\circ$$
$$= 48^\circ$$

Use trigonometric ratios to determine the missing sides.

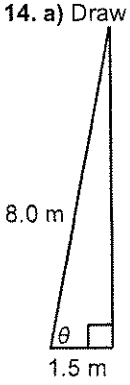
$$\sin S = \frac{13}{r}$$
$$r = \frac{13}{\sin S}$$
$$r = \frac{13}{\sin 42°}$$
$$r \doteq 19$$

Side length r is about 19 km.

$$\tan Q = \frac{q}{13}$$
$$13 \tan Q = q$$
$$13 \tan 48° = q$$
$$14 \doteq q$$

Side length q is about 14 km.

14. a) Draw a diagram to represent the situation.



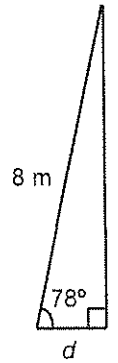
The length of the ladder is the hypotenuse of the triangle created by the ladder, the wall, and the ground. Let θ be the angle the ladder makes with the ground.

$$\cos\theta = \frac{1.5}{8.0}$$
$$\theta = \cos^{-1}\left(\frac{1.5}{8.0}\right)$$
$$\theta \doteq 79°$$

The ladder will not be stable because it is greater than 78°.

b) The greater the angle the ladder makes with the wall, the closer it is to the wall. So the minimum distance will have an angle of 78° and the maximum distance will have an angle of 73°. Let d be the distance from the ladder to the wall.

Draw a diagram for the minimum distance.



To determine the minimum distance, determine the distance d that matches cos 78°.

$$\cos 78° = \frac{d}{8.0}$$
$$8.0(\cos 78°) = 8.0\left(\frac{d}{8.0}\right)$$
$$8.0(\cos 78°) = d$$
$$d \doteq 1.7$$

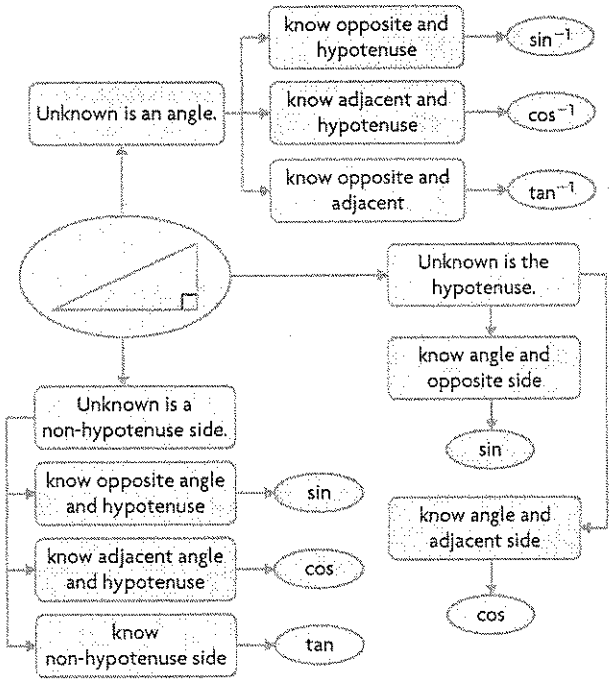
Draw a diagram of the maximum distance.

To determine the maximum distance, determine the distance d that matches cos 73°.

$$\cos 73° = \frac{d}{8.0}$$
$$8.0(\cos 73°) = 8.0\left(\frac{d}{8.0}\right)$$
$$8.0(\cos 73°) = d$$
$$d \doteq 2.3$$

The minimum safe distance is 1.7 m and the maximum safe distance is 2.3 m.

15. a) Answers will vary, e.g.,



b) Yes. If you are solving for an angle, an inverse trigonometric ratio is needed.

16. Call the 28 mm blue line C. C forms a right isosceles triangle with the two radii, r.

Use the Pythagorean theorem to determine r:

$$C^2 = r^2 + r^2$$
$$28^2 = 2r^2$$
$$784 = 2r^2$$
$$\frac{784}{2} = \frac{2r^2}{2}$$
$$392 = r^2$$
$$\sqrt{392} = r$$
$$r \doteq 19.8$$

Determine the diameter, which is twice the radius:
$d = 2r$
$d \doteq 2(19.8)$
$d \doteq 39.6$
The diameter of the circle is about 39.6 mm.
17. a) and b) Use the sine ratio to determine y:

$$\sin 30° = \frac{1}{y}$$

$$\sin 30° = \frac{1}{2}$$

So $y = 2$.
Use the Pythagorean theorem to determine x:
$y^2 = 1^2 + x^2$
$2^2 = 1^2 + x^2$
$4 = 1 + x^2$
$3 = x^2$
$x = \sqrt{3}$

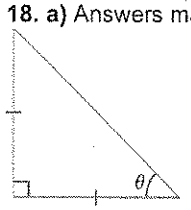
To keep x an exact value, use $\sqrt{3}$ (do not calculate the square root).

Side y is exactly 2 and side x is exactly $\sqrt{3}$.
c) Use the diagram to determine each ratio:

$$\sin 30° = \frac{1}{2} \qquad \cos 30° = \frac{\sqrt{3}}{2} \qquad \tan 30° = \frac{1}{\sqrt{3}}$$

$$\sin 60° = \frac{\sqrt{3}}{2} \qquad \cos 60° = \frac{1}{2} \qquad \tan 60° = \frac{\sqrt{3}}{1}$$

Notice that $\sin 30° = \cos 60°$, $\sin 60° = \cos 30°$, and $\tan 30° = \frac{1}{\tan 60°}$.
18. a) Answers may vary, e.g.,



b) Use the tangent ratio to determine θ:
Since this is an isosceles triangle,
$\tan\theta = 1$
$\theta = 45°$
Calculate the sine and cosine ratios for 45°:
$\sin 45° \doteq 0.7071$ $\cos 45° \doteq 0.7071$
In any right isosceles triangle, $\tan\theta = 1$ and $\sin\theta = \cos\theta$. This makes sense because the side opposite θ and the side adjacent to θ are equal.

Curious Math, page 407

1. a) Determine 16' as a decimal number of degrees:
Since 1 minute is $\frac{1}{60}$ of a degree, then,
16 minutes is $\frac{16}{60}$ of a degree, or about 0.27°.
That means 25°16' is about 25.27°.
b) Determine 0.30° as a number of minutes:
Let m be the number of minutes.
If 1 minute is $\frac{1}{60}$ of a degree, then

$$m\left(\frac{1}{60}\right) = 0.30°$$

$$60\left(\frac{m}{60}\right) = 60(0.30°)$$

$$m = 18$$

0.30° = 18'
That means 48.30° is 48°18'.
2. a) Converting 44°15' to a decimal:
15 minutes is $\frac{15}{60}$ of a degree, or 0.25°, so 44°15' is 44.25°.
Substitute $\theta = 44.25°$, into the given formula to calculate M:
$M = 1852.27 - 9.45(\cos 2\theta)$
$M = 1852.27 - 9.45[\cos 2(44.25°)]$
$M = 1852.27 - 9.45(\cos 88.5°)$
$M \doteq 1852.02 - 9.45(0.2618)$
$M \doteq 1852.27 - 2.4740$
$M \doteq 1852.02$
The nautical mile at Kingston is about 1852.02 m long.
b) Converting from degrees and minutes into decimals, 28 minutes is $\frac{28}{60}$ of a degree, or about 0.47°. So, 62°28' is about 62.47°. Setting $\theta = 62.47°$, we can use the given formula to calculate length.
$M = 1852.27 - 9.45(\cos 2\theta)$
$M = 1852.27 - 9.45[\cos 2(62.47°)]$
$M = 1852.27 - 9.45(\cos 124.54°)$
$M \doteq 1857.63$
Therefore, the nautical mile at Yellowknife is about 1857.63 m long.
c) Converting from degrees and minutes into decimals, 30 minutes is $\frac{30}{60}$ of a degree, or 0.5°. So 82°30' is 82.5°.
Setting $\theta = 82.5°$, we can use the given formula to calculate length.
$M = 1852.27 - 9.45(\cos 2\theta)$
$M = 1852.27 - 9.45[\cos 2(82.5°)]$
$M = 1852.27 - 9.45(\cos 165°)$
$M \doteq 1861.40$
Therefore, the nautical mile at Alert is about 1861.40 m.
3. This value was chosen because the expression $9.45\cos 2\theta$ is small compared with 1852, so most of the values for M are approximately 1852 m. It is approximately the mean length of one minute of latitude, that is, one minute of arc along a line of longitude.
4. The expression $9.45(\cos 2\theta)$ represents the change in the distance of the nautical mile depending on the latitude.

Lesson 7.6: Solving Right Triangle Problems, page 412

1. Use the sine ratio and solve for the hypotenuse.

$$\sin 50^\circ = \frac{15-1}{h}$$

$$h = \frac{14}{\sin 50^\circ}$$

$$h \doteq 18$$

She has released 18 m of string.

2. Use the cosine ratio and solve for the hypotenuse.

$$\cos 76^\circ = \frac{0.8}{h}$$

$$h = \frac{0.8}{\cos 76^\circ}$$

$$h \doteq 3.3$$

Subtract h from the total length of the ladder:

$6 - 3.3 = 2.7$

Bill was about 2.7 m from the top of the ladder.

3. Use the sine ratio and solve for the angle.

Let θ represent the angle formed by the wire and the ground.

$$\sin\theta = \frac{24}{30}$$

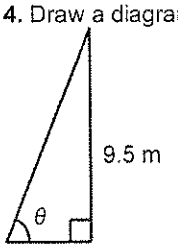
$$\sin\theta = 0.8$$

$$\theta = \sin^{-1}(0.8)$$

$$\theta \doteq 53^\circ$$

The angle formed by the guy wire and the ground is about 53°.

4. Draw a diagram to represent the situation.



Use the tangent ratio and solve for the angle.

$$\tan\theta = \frac{9.5}{3.8}$$

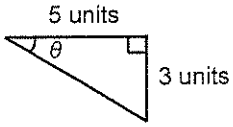
$$\tan\theta = 2.5$$

$$\theta = \tan^{-1}(2.5)$$

$$\theta \doteq 68^\circ$$

The sun is at an angle of elevation of about 68°.

5. Draw a diagram to represent the situation.



Use the tangent ratio and solve for the angle.

$$\tan\theta = \frac{3}{5}$$

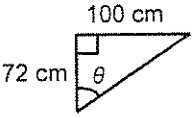
$$\tan\theta = 0.6$$

$$\theta = \tan^{-1}(0.6)$$

$$\theta \doteq 31^\circ$$

The angle of depression of the rafter is about 31°.

6. Draw a diagram to represent the situation.



Use the tangent ratio and solve for the angle.

$$\tan\theta = \frac{72}{100}$$

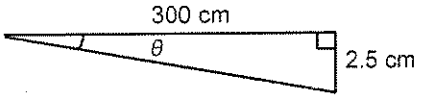
$$\tan\theta = 0.72$$

$$\theta = \tan^{-1}(0.72)$$

$$\theta \doteq 36^\circ$$

The maximum angle at which the stairs can rise is about 36°.

7. Draw a diagram to represent the situation.



Use the tangent ratio and solve for the angle.

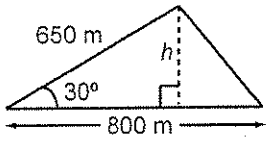
$$\tan\theta = \frac{2.5}{300}$$

$$\theta = \tan^{-1}\left(\frac{2.5}{300}\right)$$

$$\theta \doteq 0.5^\circ$$

The contractor should lay the pipe at about a 0.5° angle.

8. Draw a diagram to represent the situation.



Use the sine ratio and solve for the height of the triangle.

$$\sin 30^\circ = \frac{h}{650}$$

$$650\sin 30^\circ = h$$

$$325 = h$$

Calculate the area of the triangle.

Area $= 0.5(800)(325)$

$= 130\,000$

The area enclosed by the trenches is 130 000 m^2.

9. Use the tangent ratio and solve for the angle.

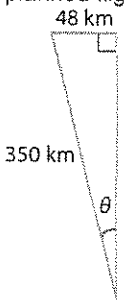
$$\tan\theta = \frac{64}{71}$$

$$\theta = \tan^{-1}\left(\frac{64}{71}\right)$$

$$\theta \doteq 42^\circ$$

The stairs rise at an angle of about 42°.

10. Use the sine ratio and solve for the angle. Let θ be the angle between the actual flight path and the planned flight path.



$$\sin\theta = \frac{48}{350}$$

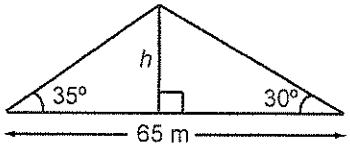
$$\theta = \sin^{-1}\left(\frac{48}{350}\right)$$

$$\theta \doteq 8^\circ$$

The airplane is about 8° off its planned flight path.

11. The tree divides the triangle into two right triangles.

Draw a diagram to represent the situation.



The height of the tree, h, is the opposite side for each of the given angles, so the side that is adjacent to each angle can be represented using the tangent ratio for each angle:

The side adjacent to 35° is $\frac{h}{\tan}\times 35°$.

The side adjacent to 30° is $\frac{h}{\tan}\times 30°$.

Since the total length is 65 m, we can write the following equation and then solve for h:

$$65 = \frac{h}{\tan 35^\circ} + \frac{h}{\tan 30^\circ}$$

$$65 \doteq \frac{h}{0.7} + \frac{h}{0.6}$$

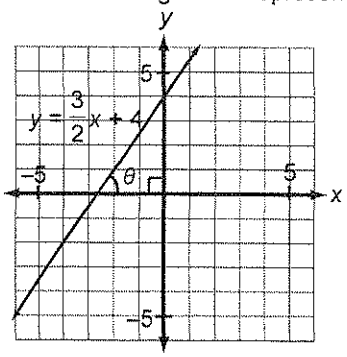
$$27.3 \doteq 0.6h + 0.7h$$

$$27.3 \doteq 1.3h$$

$$21 \doteq h$$

The tree is about 21 m tall.

12. Draw a diagram to represent the situation.



Since the equation is in the form $y = mx + b$, it's easy to see that the slope $\left(\frac{\text{rise}}{\text{run}}\right)$ is $\frac{3}{2}$. That means I can use the tangent ratio and then solve for the angle.

$$\tan\theta = \frac{3}{2}$$

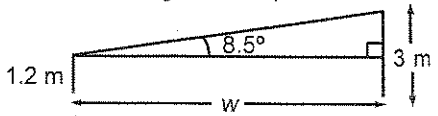
$$\tan\theta = 1.5$$

$$\theta = \tan^{-1}(1.5)$$

$$\theta \doteq 56^\circ$$

The angle between the line and the x-axis is about 56°.

13. Draw a diagram to represent the situation.



Use the tangent ratio and solve for the width of the river.

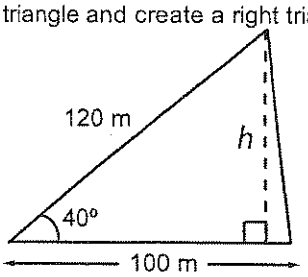
$$\tan 8.5^\circ = \frac{3-1.2}{w}$$

$$w = \frac{1.8}{\tan 8.5^\circ}$$

$$w \doteq 12.0$$

The river is about 12.0 m wide.

14. Answers may vary, e.g., I would first draw a line from the base to the topmost vertex to represent the height of the triangle and create a right triangle.



Then I would calculate the height of the triangle, h, using $\sin 40^\circ = \frac{h}{120}$. Next, I would determine the area of the triangle in square metres using $A = 0.5(100)(h)$. Finally, I would multiply the area by \$1.50/m^2, the cost of sod per square metre.

15. a) Use the tangent ratio and solve for the distance between the buildings.

$$\tan 36^\circ = \frac{120}{d}$$

$$d = \frac{120}{\tan 36^\circ}$$

$$d \doteq 165$$

The buildings are 165 m apart.

b) Use the tangent ratio and solve for the unknown height.

$$\tan 47^\circ = \frac{h}{165}$$

$$165 \tan 47^\circ = h$$

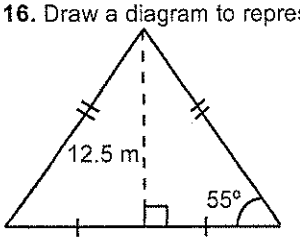
$$177 = h$$

Add the heights:

177 + 120 = 297

The building is 297 m tall.

16. Draw a diagram to represent the situation.



Use the tangent ratio and solve for the adjacent side, which is half the base.

$$\tan 55^\circ = \frac{12.5}{a}$$

$$a = \frac{12.5}{\tan 55^\circ}$$

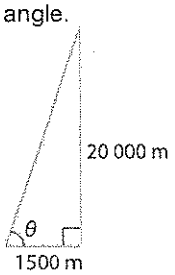
$$a \doteq 8.75$$

Area $= 0.5(2 \times 8.75)(12.5)$

$= 109.4$

The area is 109.4 m^2.

17. Let θ be the angle at which Lucien needs to set his camera. Use the tangent ratio and solve for the angle.



$$\tan\theta = \frac{20\,000}{1500}$$

$$\theta = \tan^{-1}\left(\frac{20\,000}{1500}\right)$$

$$\theta \doteq 86^\circ$$

Lucien should set his camera at an angle of about 86°.

18. Answers may vary, e.g.,

Draw a diagram and label it with any given information.

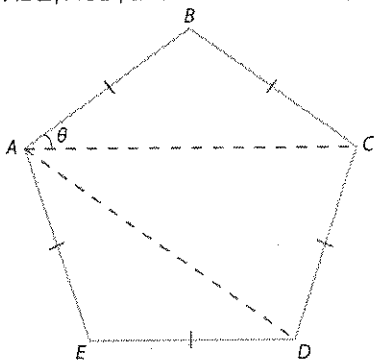
If two sides are given, use the Pythagorean theorem to determine the third side.

If one acute angle is given, calculate the third angle measure using the fact that the sum of the interior angles is 180°.

To solve for a side, use the appropriate trigonometric ratio.

To solve for an angle, use the appropriate inverse trigonometric ratio.

19. a) Figure out the total of the regular pentagon's interior angles by dividing it into three triangles: *ADE*, *ACD*, and *ABC*.



The total of the interior angles of each triangle is 180°.

The total of the interior angles of the pentagon is equal to the sum of the interior angles of three triangles: $3 \times 180^\circ = 540^\circ$.

Find each interior angle by dividing by 5:

$540^\circ \div 5 = 108^\circ$

Since each interior angle in the regular pentagon is 108° and $\triangle ABC$ is an isosceles triangle, then:

$$180^\circ = 108^\circ + \theta + \theta$$

$$180^\circ - 108^\circ = 2\theta$$

$$72^\circ = 2\theta$$

$$36^\circ = \theta$$

b) Use the cosine ratio and solve for half of *AC*, *x*.

$$\cos 36^\circ = \frac{x}{8.2}$$

$$8.2 \cos 36^\circ = x$$

$$6.63 \doteq x$$

Half of *AC*, *x*, is 6.63 cm long.

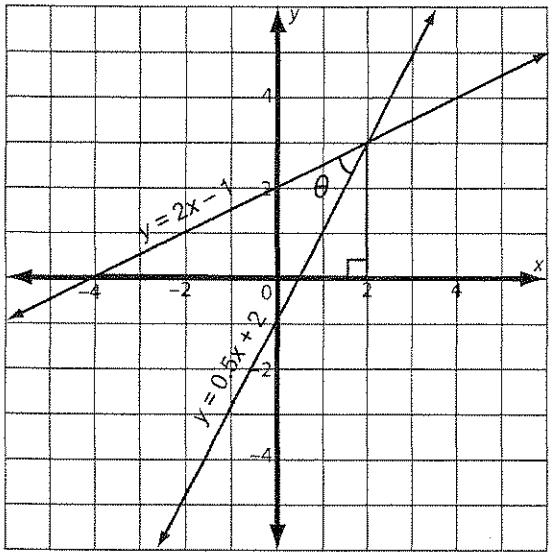
Calculate the length of *AC*.

$AC = 2(6.63)$

$= 13.3$

Diagonal *AC* is 13.3 cm long.

20. If you graph the two lines, two right angle triangles are created. One has vertices at (2, 3), (–4, 0), and (2, 0) and the other has vertices at (2, 3), (0.5, 0), (2, 0). Let θ represent the acute angle between the two lines.



Call the top angle of the large triangle $\angle A$ and the top angle of the small triangle $\angle$ ($\angle B$ is included in $\angle A$). The large triangle has a base of length 6 and a height of length 3. The small triangle has a base of length 1.5 and a height of 3.

Use the tangent ratio to solve for $\angle A$ and $\angle B$.

$$\tan A = \frac{6}{3} \qquad \tan B = \frac{1.5}{3}$$

$$\angle A = \tan^{-1}\left(\frac{6}{3}\right) \qquad \angle B = \tan^{-1}\left(\frac{1.5}{3}\right)$$

$$\angle A \doteq 63^\circ \qquad \angle B \doteq 26^\circ$$

Since $\angle A$ is composed of $\angle B$ and θ, subtract $\angle B$ from $\angle A$:

$$\angle A = \angle B + \theta$$
$$\angle A - \angle B = \theta$$
$$63^\circ - 26^\circ \doteq \theta$$
$$\theta \doteq 37^\circ$$

The two lines intersect at an angle of about 37°.

Chapter Review, page 416

1. Yes, they are similar because all the corresponding pairs of angles are equal. To write a proportion statement and calculate the scale factor, create ratios using the corresponding sides:

$\frac{9}{4} = \frac{11.25}{5} = \frac{6.75}{3} = 2.25$. A scale factor of 2.25 means each side in the larger triangle is 2.25 times the length of the corresponding side in the smaller triangle.

2. The triangles are similar because there are two equal corresponding angles ($\angle ACB = \angle ECD$) because they are opposite angles and $\angle B = \angle D = 90^\circ$). The ratios of the corresponding side lengths of similar triangles are the same, so create a proportion and solve it for p.

$$\frac{4}{6} = \frac{p}{5}$$
$$5\left(\frac{4}{6}\right) = 5\left(\frac{p}{5}\right)$$
$$5\left(\frac{4}{6}\right) = p$$
$$p \doteq 3$$

Side p is about 3 cm long.

3. This ramp is composed of three triangles. The largest triangle has a base length 0.9 + 1.2 + 1.7 = 3.8 m, the medium sized triangle has a base length 1.2 + 1.7 = 2.9 m, and the smallest triangle has a base length of 1.7 m. Since each triangle is a right angle, and they all share a common acute angle in the bottom corner, all three triangles are similar to each other. Since they are similar, the ratios of the corresponding side lengths are the same.

Create a proportion for the medium and large triangles and solve for x:

$$\frac{2.9}{3.8} = \frac{x}{1.3}$$
$$1.3\left(\frac{2.9}{3.8}\right) = 1.3\left(\frac{x}{1.3}\right)$$
$$1.3\left(\frac{2.9}{3.8}\right) = x$$
$$x \doteq 1.0$$

Create a proportion for the small and large triangles and solve for y:

$$\frac{1.7}{3.8} = \frac{y}{1.3}$$
$$1.3\left(\frac{1.7}{3.8}\right) = 1.3\left(\frac{y}{1.3}\right)$$
$$1.3\left(\frac{1.7}{3.8}\right) = y$$
$$y \doteq 0.6$$

Therefore, x is about 1 m tall and y is about 0.6 m tall.

4. Since the angle of elevation of the sun will be the same for Brett and the radio tower and they are both perpendicular to the ground, the two triangles formed by Brett and the tower have two equal corresponding angles making them similar triangles. Since they are similar, the ratios of their corresponding side lengths are the same. Determine the height of the tower. Let h represent the height of the tower, write a proportion using corresponding side lengths, and then solve for h.

$$\frac{9.00}{0.60} = \frac{h}{1.85}$$
$$1.85\left(\frac{9.00}{0.60}\right) = 1.85\left(\frac{h}{1.85}\right)$$
$$1.85\left(\frac{9.00}{0.60}\right) = h$$
$$h = 27.75$$

The height of the tower is 27.75 m.
Determine the length of the guy wire.
The length of the guy wire is the hypotenuse of the triangle formed by the wire, the tower, and its shadow. Let l represent the length of the guy wire, use the Pythagorean Theorem, and solve for l.

$$l^2 = h^2 + 9.00^2$$
$$l^2 = 27.75^2 + 9.00^2$$
$$l = \sqrt{27.75^2 + 9.00^2}$$
$$l \doteq \sqrt{770.06 + 81.00}$$
$$l \doteq 29.17$$

Brett will need about 29.17 m of guy wire.

5. a) First determine the length of the hypotenuse. Let l represent the hypotenuse and use the Pythagorean theorem to calculate l.

$$l^2 = 4^2 + 8^2$$
$$l = \sqrt{4^2 + 8^2}$$
$$l = \sqrt{16 + 64}$$
$$l = \sqrt{80}$$
$$l = 9$$

The hypotenuse is 9 cm.
Use the side lengths to write the ratios.

$$\sin A \doteq \frac{4}{9} \quad \cos A \doteq \frac{8}{9} \quad \tan A = \frac{4}{8} = \frac{1}{2}$$

b) Using the tangent ratio.

$$\tan A = \frac{1}{2}$$
$$\angle A = \tan^{-1}\left(\frac{1}{2}\right)$$
$$\angle A \doteq 27°$$

6. a)
$$\tan 46° = \frac{x}{14.2}$$
$$14.2(\tan 46°) = 14.2\left(\frac{x}{14.2}\right)$$
$$14.2(\tan 46°) = x$$
$$x \doteq 14.7$$

Side length x is about 14.7 units.

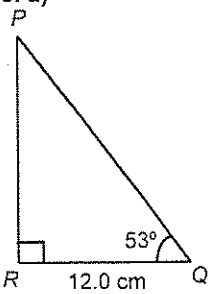
b)
$$\cos 29° = \frac{17.3}{x}$$
$$x(\cos 29°) = x\left(\frac{17.3}{x}\right)$$
$$x(\cos 29°) = 17.3$$
$$\frac{x(\cos 29°)}{\cos 29°} = \frac{17.3}{\cos 29°}$$
$$x = \frac{17.3}{\cos 29°}$$
$$x \doteq 19.8$$

Side length x is about 19.8 units.

7. In $\triangle ABC$, BC is opposite $\angle BAC$ and AB is adjacent to $\angle BAC$. So use the tangent ratio to determine $\angle BAC$.

$$\tan \angle BAC = \frac{BC}{AB}$$
$$\tan \angle BAC = \frac{10}{15}$$
$$\angle BAC = \tan^{-1}\left(\frac{10}{15}\right)$$
$$\angle BAC \doteq 34°$$

8. a)

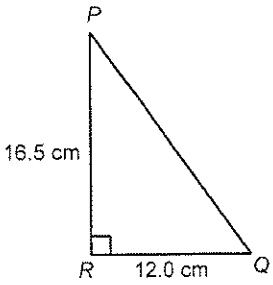


$\triangle PQR$ is a right triangle. Side r is the hypotenuse because it is opposite the right angle, $\angle R$. The side that is adjacent to $\angle Q$, 53°, is p, which is 12.0 cm.
Use the cosine ratio to determine r.

$$\cos Q = \frac{p}{r}$$
$$\cos 53° = \frac{12.0}{r}$$
$$r(\cos 53°) = r\left(\frac{12.0}{r}\right)$$
$$r(\cos 53°) = 12.0$$
$$\frac{r(\cos 53°)}{\cos 53°} = \frac{12.0}{\cos 53°}$$
$$r = \frac{12.0}{\cos 53°}$$
$$r \doteq 20$$

Side r is about 20 cm.

b)



$\triangle PQR$ is a right triangle. Side r is the hypotenuse because it is opposite the right angle, $\angle R$. The side that is adjacent to $\angle P$ is q, which is 16.5 cm, and the side that is opposite $\angle P$ is p, which is 12.0 cm.

Use the tangent ratio to determine $\angle P$.

$$\tan P = \frac{p}{q}$$
$$\tan P = \frac{12.0}{16.5}$$
$$\angle P = \tan^{-1}\left(\frac{12.0}{16.5}\right)$$
$$\angle P \doteq 36.0°$$

$\angle P$ is about 36.0°

9. Determine θ using the fact that the sum of the interior angles of a triangle must be 180°.

$$\theta + 90° + 18° = 180°$$
$$\theta = 180° - 90° - 18°$$
$$\theta = 72°$$

Use the sine ratio to solve for a:

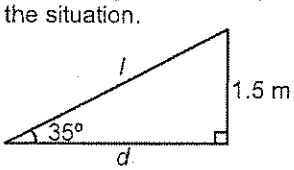
$$\sin 18° = \frac{9}{a}$$
$$a(\sin 18°) = a\left(\frac{9}{a}\right)$$
$$a(\sin 18°) = 9$$
$$\frac{a(\sin 18°)}{\sin 18°} = \frac{9}{\sin 18°}$$
$$a = \frac{9}{\sin 18°}$$
$$a \doteq 29$$

Use the tangent ratio to solve for b:

$$\tan 18° = \frac{9}{b}$$
$$b(\tan 18°) = b\left(\frac{9}{b}\right)$$
$$b(\tan 18°) = b$$
$$\frac{b(\tan 18°)}{\tan 18°} = \frac{9}{\tan 18°}$$
$$b = \frac{9}{\tan 18°}$$
$$b \doteq 28$$

θ is 72°, a is about 29 cm, and b is about 28 cm.

10. Let d be the distance between where the ramp touches the ground and the rear of the truck and let l be the length of the ramp. Draw a picture to represent the situation.



a) Side d is adjacent to the 35° angle, and the 1.5 m side is opposite the 35° angle. Use the tangent ratio to solve for d.

$$\tan 35° = \frac{1.5}{d}$$
$$d(\tan 35°) = d\left(\frac{1.5}{d}\right)$$
$$d(\tan 35°) = 1.5$$
$$\frac{d(\tan 35°)}{\tan 35°} = \frac{1.5}{\tan 35°}$$
$$d \doteq 2.1$$

The distance between where the ramp touches the ground and the rear of the truck is about 2.1 m

b) Side l is opposite the right angle, so it is the hypotenuse. Use the other two known side lengths and the Pythagorean theorem to determine l:

$$l^2 = d^2 + 1.5^2$$
$$l^2 \doteq 2.1^2 + 1.5^2$$
$$l \doteq \sqrt{2.1^2 + 1.5^2}$$
$$l \doteq \sqrt{4.41 + 2.25}$$
$$l \doteq \sqrt{6.66}$$
$$l \doteq 2.6$$

Therefore, the ramp should be 2.6 m.

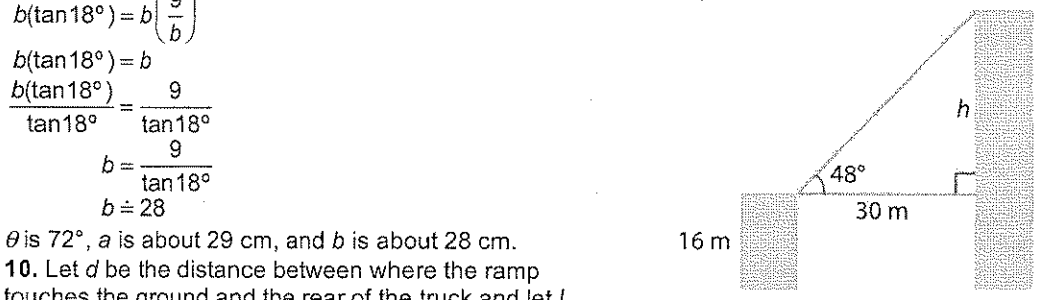
11. The distance the airplane must travel, d, is the length of the side adjacent to the 12° angle. The length of the side opposite to the 12° angle is 1200 m.

Use the tangent ratio to solve for d:

$$\tan 12° = \frac{1200}{d}$$
$$d(\tan 12°) = d\left(\frac{1200}{d}\right)$$
$$d(\tan 12°) = 1200$$
$$\frac{d(\tan 12°)}{\tan 12°} = \frac{1200}{\tan 12°}$$
$$d = \frac{1200}{\tan 12°}$$
$$d \doteq 5646$$

The airplane must travel about 5646 m.

12. Draw a diagram to represent the situation. Let h represent the difference in the buildings' heights.



The distance between the buildings is the side adjacent to the 48° angle and h is the side opposite the 48° angle. Use the tangent ratio to solve for h.

$$\tan 48° = \frac{h}{30}$$
$$30(\tan 48°) = 30\left(\frac{h}{30}\right)$$
$$30(\tan 48°) = h$$
$$h \doteq 33$$

The second building is about 33 m taller than the first building. Since the first building is 16 m tall, the second building is about 33 + 16 = 49 m tall.

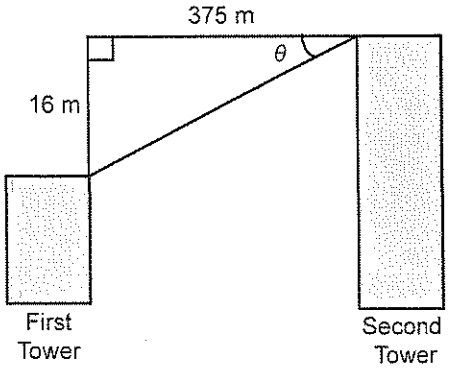
13. Let θ be the angle of elevation. Draw a diagram to represent the situation.

The vertical distance is opposite the angle of elevation, θ, and the distance the cyclist pedals is the hypotenuse. Use the sine ratio to calculate θ.

$$\sin\theta = \frac{1.1}{6.5}$$
$$\theta = \sin^{-1}\left(\frac{1.1}{6.5}\right)$$
$$\theta \doteq 10°$$

The angle of elevation is about 10°.

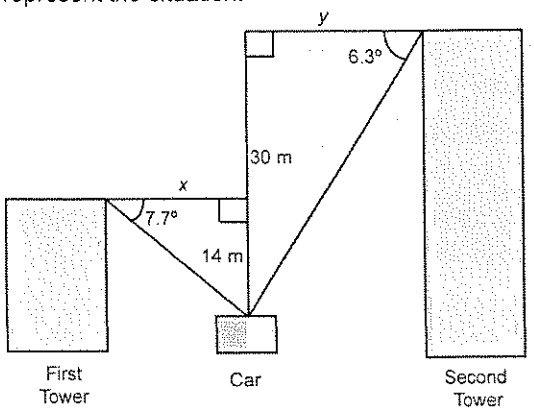
14. a) Let θ be the angle of depression. The top of the second tower is 30 – 14 = 16 m above the top of the first tower. Draw a diagram to show this.



Use the tangent ratio to calculate θ.

$$\tan\theta = \frac{16}{375}$$
$$\theta = \tan^{-1}\left(\frac{16}{375}\right)$$
$$\theta \doteq 2°$$

b) For each of the two triangles formed, the distance between the car and the base of each tower is adjacent to the angle of depression. Let x be the distance between the car and the base of the first tower and y be the distance between the car and the base of the second tower. Draw a diagram to represent the situation.



Use the tangent ratio to solve for x and then for y.

For the first tower:

$$\tan 7.7° = \frac{14}{x}$$
$$x(\tan 7.7°) = x\left(\frac{14}{x}\right)$$
$$x(\tan 7.7°) = 14$$
$$\frac{x(\tan 7.7°)}{\tan 7.7°} = \frac{14}{\tan 7.7°}$$
$$x = \frac{14}{\tan 7.7°}$$
$$x \doteq 104$$

For the second tower:

$$\tan 6.3° = \frac{30}{y}$$
$$y(\tan 6.3°) = y\left(\frac{30}{y}\right)$$
$$y(\tan 6.3°) = 30$$
$$\frac{y(\tan 6.3°)}{\tan 6.3°} = \frac{30}{\tan 6.3°}$$
$$y = \frac{30}{\tan 6.3°}$$
$$y \doteq 272$$

The guard in the first tower is closer to the car.

15. Determine AD and BD and then determine the difference, AB.

Solving for AD:

$$\tan 40° = \frac{AD}{CD}$$
$$\tan 40° = \frac{AD}{100}$$
$$100(\tan 40°) = 100\left(\frac{AD}{100}\right)$$
$$100(\tan 40°) = AD$$
$$AD \doteq 84$$

Solving for BD:

$$\tan 20° = \frac{BD}{CD}$$
$$\tan 20° = \frac{BD}{100}$$
$$100(\tan 20°) = 100\left(\frac{BD}{100}\right)$$
$$100(\tan 20°) = BD$$
$$BD \doteq 36$$

Calculate the difference to determine AB:

$$AB = AD - BD$$
$$\doteq 84 - 36$$
$$\doteq 48$$

AB is about 48 m long.

16. Determine AC and BC and then determine the difference, AB.

Solving for AC

$$\tan 30° = \frac{70}{AC}$$
$$AC(\tan 30°) = AC\left(\frac{70}{AC}\right)$$
$$AC(\tan 30°) = 70$$
$$\frac{AC(\tan 30°)}{\tan 30°} = \frac{70}{\tan 30°}$$
$$AC = \frac{70}{\tan 30°}$$
$$AC \doteq 121$$

Solving for BC:

$$\tan 45° = \frac{70}{BC}$$
$$BC(\tan 45°) = BC\left(\frac{70}{BC}\right)$$
$$BC(\tan 45°) = 70$$
$$\frac{BC(\tan 45°)}{\tan 45°} = \frac{70}{\tan 45°}$$
$$BC = \frac{70}{\tan 45°}$$
$$BC = 70$$

Calculate the difference to determine AB:

$$AB = AC - BC$$
$$\doteq 121 - 70$$
$$\doteq 51$$

The swimmer swam about 51 m.

17. The hypotenuse of the triangle formed is the distance the plane travelled in 10 s at 60 m/s. Since distance is speed × time, this is 60 m/s × 10 s = 600 m. The height of the plane, 300 m, is opposite the angle of elevation, θ.

Use the sine ratio to calculate θ.

$$\sin\theta = \frac{300}{600}$$
$$\theta = \sin^{-1}\left(\frac{300}{600}\right)$$
$$\theta = 30°$$

The angle of elevation is 30°.

Chapter Self-Test, page 418

1. First triangle:

From the diagram you can see that a = 11.80.

Side a is 11.80 units.

Second triangle:

Since both triangles have two equal corresponding angles, they are similar triangles. That means corresponding sides are in proportion.

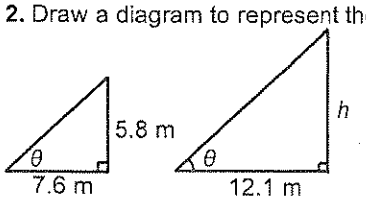
Write a proportion and then solve it for b:

$$\frac{16.48}{10.30} = \frac{b}{11.80}$$
$$11.80\left(\frac{16.48}{10.30}\right) = 11.80\left(\frac{b}{11.80}\right)$$
$$11.80\left(\frac{16.48}{10.30}\right) = b$$
$$b = 18.88$$

From the diagram, you can see that c = b, so c = 18.88.

Sides b and c are 18.88 units.

2. Draw a diagram to represent the situation.



Since both trees are perpendicular to the ground and the angle of elevation, θ, is the same, the triangles have two equal corresponding angles, making them similar triangles. That means corresponding side lengths are in proportion.

Let h represent the height of the taller tree. Write a proportion and then solve it for h.

$$\frac{12.1}{7.6} = \frac{h}{5.8}$$
$$5.8\left(\frac{12.1}{7.6}\right) = h$$
$$h \doteq 9.2$$

The height of the taller tree is 9.2 m.

3. a) $\sin 28° = \frac{x}{5}$

$$5(\sin 28°) = 5\left(\frac{x}{5}\right)$$
$$5(\sin 28°) = x$$
$$x \doteq 2.3$$

b) $\cos 43° = \frac{13}{y}$

$$y(\cos 43°) = y\left(\frac{13}{y}\right)$$
$$y(\cos 43°) = 13$$
$$y = \frac{13}{\cos 43°}$$
$$y \doteq 17.8$$

c) $\tan A = 7.1154$

$$\angle A = \tan^{-1}(7.1154)$$
$$\angle A \doteq 82.0°$$

d) $\cos B = \frac{7}{9}$

$$\angle B = \cos^{-1}\left(\frac{7}{9}\right)$$
$$\angle B \doteq 38.9°$$

4. a) Since a is opposite the right angle, a is the hypotenuse of the right triangle. The 50 cm side length is opposite the 73° angle.

Use the sine ratio for 73° to solve for a.

$$\sin 73° = \frac{50}{a}$$
$$a\sin 73° = a\left(\frac{50}{a}\right)$$
$$a\sin 73° = 50$$
$$\frac{a\sin 73°}{\sin 73°} = \frac{50}{\sin 73°}$$
$$a = \frac{50}{\sin 73°}$$
$$a \doteq 52$$

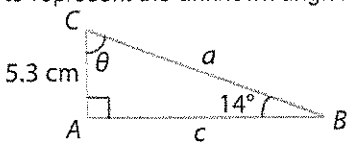
Side a is about 52 cm.

b) Since the 38 cm side is opposite the right angle, it is the hypotenuse of the right triangle. The 15 cm side length is adjacent to angle θ.

Use the cosine ratio to solve for θ.

$$\cos\theta = \frac{15}{38}$$
$$\theta = \cos^{-1}\left(\frac{15}{38}\right)$$
$$\theta \doteq 67°$$

5. a) Draw a diagram to represent the situation. Use θ to represent the unknown angle.



Use the fact that the sum of the interior angles of a triangle must be 180° to determine $\angle C$.

$\angle A + \angle B + \angle C = 180°$

$90° + 14° + \angle C = 180°$

$\angle C = 180° - 90° - 14°$

$\angle C = 76°$

Determine side length a.

Side a is opposite the right angle, so it is the hypotenuse. Side b which is 5.3 cm, is opposite $\angle B$.

Use the sine ratio for 14° to solve for a.

$$\sin B = \frac{b}{a}$$

$$\sin 14° = \frac{5.3}{a}$$

$$a(\sin 14°) = a\left(\frac{5.3}{a}\right)$$

$$a(\sin 14°) = 5.3$$

$$\frac{a(\sin 14°)}{\sin 14°} = \frac{5.3}{\sin 14°}$$

$$a = \frac{5.3}{\sin 14°}$$

$$a \doteq 21.9$$

Side a is about 21.9 cm.

Determine side length c.

Side c is adjacent to $\angle B$. Side b, 5.3 cm, is opposite $\angle B$. Use the tangent ratio for 14° to solve for c.

$$\tan B = \frac{b}{c}$$

$$\tan 14° = \frac{5.3}{c}$$

$$c(\tan 14°) = c\left(\frac{5.3}{c}\right)$$

$$c(\tan 14°) = 5.3$$

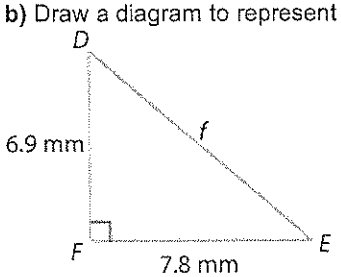
$$\frac{c(\tan 14°)}{\tan 14°} = \frac{5.3}{\tan 14°}$$

$$c = \frac{5.3}{\tan 14°}$$

$$c \doteq 21.3$$

Side c is about 21.3 cm.

b) Draw a diagram to represent the situation.



Use the Pythagorean theorem to calculate f.

$f^2 = d^2 + e^2$

$f^2 = 7.8^2 + 6.9^2$

$f = \sqrt{7.8^2 + 6.9^2}$

$f = \sqrt{108.45}$

$f \doteq 10.4$

Side f is about 10.4 mm.

Determine $\angle D$.

Side d, 7.8 mm, is opposite $\angle D$ and side e, 6.9 mm, is adjacent to $\angle D$.

Use the tangent ratio to solve for $\angle D$.

$$\tan D = \frac{d}{e}$$

$$\tan D = \frac{7.8}{6.9}$$

$$\angle D = \tan^{-1}\left(\frac{7.8}{6.9}\right)$$

$$\angle D \doteq 49°$$

Determine $\angle E$.

Side e which is 6.9 mm, is opposite $\angle E$ and side d, which is 7.8 mm, is adjacent to $\angle E$.

Use the tangent ratio to solve for $\angle E$.

$$\tan E = \frac{e}{d}$$

$$\tan E = \frac{6.9}{7.8}$$

$$\angle E = \tan^{-1}\left(\frac{6.9}{7.8}\right)$$

$$\angle E \doteq 41°$$

6. Determine the length of the ramp.

The length of the ramp is the hypotenuse. The rise of the ramp is opposite the 4.8° angle.

Let l represent the length of the ramp. Use a sine ratio of 4.8° to solve for l.

$$\sin 4.8° = \frac{1.20}{l}$$

$$l(\sin 4.8°) = l\left(\frac{1.20}{l}\right)$$

$$l(\sin 4.8°) = 1.20$$

$$l(\sin 4.8°) = \frac{1.20}{\sin 4.8°}$$

$$l = \frac{1.20}{\sin 4.8°}$$

$$l \doteq 14.34$$

The length of the ramp is 14.34 m.

Determine the length of the run.

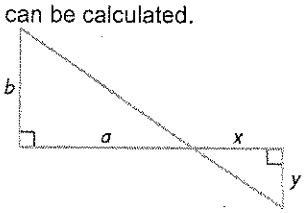
The run of the ramp is adjacent to the 4.8° angle. The rise of the ramp is opposite the 4.8° angle.

Let r represent the run of the ramp. Use the tangent ratio of 48° to solve for r.

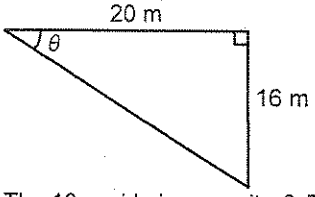
$$\tan 4.8° = \frac{1.20}{r}$$
$$r(\tan 4.8°) = r\left(\frac{1.20}{r}\right)$$
$$r(\tan 4.8°) = 1.20$$
$$\frac{r(\tan 4.8°)}{\tan 4.8°} = \frac{1.20}{\tan 4.8°}$$
$$r = \frac{1.20}{\tan 4.8°}$$
$$r \doteq 14.29$$

The run of the ramp is about 14.29 m.

7. Answers may vary, e.g., let the width of the river be b. The surveyors are standing at the point where the two triangles meet. Let a be the distance from the surveyors to one point along the river's edge, and let x be the distance from the surveyors to another point along the river's edge. Let y be the distance from the surveyors to a point along the land. Since two pairs of corresponding angles are congruent, the two triangles formed are similar triangles. If the surveyors can measure the values of a, x, and y, then the value of b can be calculated.



8. Draw a diagram to represent the situation. Let θ be the angle of depression.

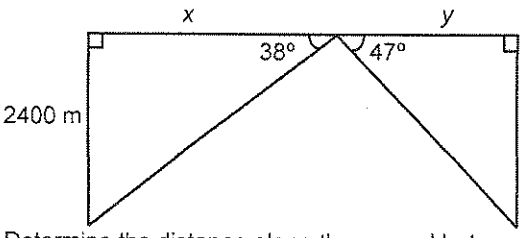


The 16 m side is opposite θ. The 20 m side is adjacent to θ. Use a tangent ratio to solve for θ.

$$\tan\theta = \frac{16}{20}$$
$$\theta = \tan^{-1}\left(\frac{16}{20}\right)$$
$$\theta \doteq 39°$$

The angle of depression is 39°.

9. Draw a diagram to represent the situation. Let x represent the distance along the ground between the plane and the fire to the west, and y represent the distance along the ground between the plane and the fire to the east.



Determine the distance along the ground between the plane and the fire to the west.

Side x is adjacent to the 38° angle. The side opposite to the 38° angle is the altitude of the plane, 2400 m.

Use the tangent ratio of 38° to solve for x.

$$\tan 38° = \frac{2400}{x}$$
$$x(\tan 38°) = x\left(\frac{2400}{x}\right)$$
$$x(\tan 38°) = 2400$$
$$\frac{x(\tan 38°)}{\tan 38°} = \frac{2400}{\tan 38°}$$
$$x = \frac{2400}{\tan 38°}$$
$$x \doteq 3072$$

The distance along the ground between the plane and the fire to the west is about 3072 m.

Determine the distance along the ground between the plane and the fire to the east.

Side y is adjacent to the 48° angle. The side opposite the 48° angle is the altitude of the plane, 2400 m. Use the tangent ratio to solve for y.

$$\tan 47° = \frac{2400}{y}$$
$$y(\tan 47°) = y\left(\frac{2400}{y}\right)$$
$$y(\tan 47°) = 2400$$
$$\frac{y(\tan 47°)}{\tan 47°} = \frac{2400}{\tan 47°}$$
$$y = \frac{2400}{\tan 47°}$$
$$y \doteq 2238$$

The distance along the ground between the plane and the fire to the east is 2238 m.

Add the two distances to determine the distance between the fires.

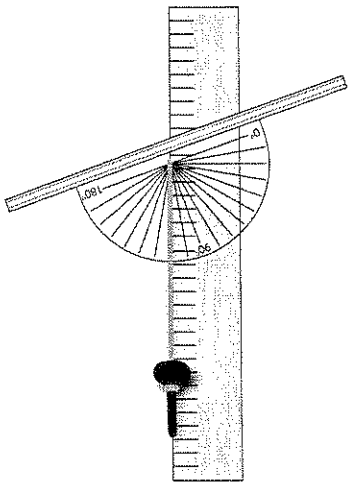
3072 + 2238 = 5310

The distance between the fires is 5310 m.

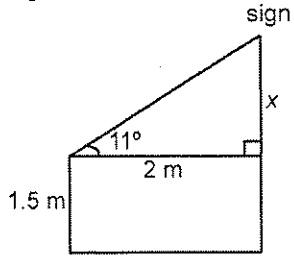
Chapter Task, page 419

Answers may vary, e.g.,

A. I started by looking at the picture of a clinometer on page 409 of the text. To make a clinometer, I will tape the drinking straw along the straight edge of the protractor. I will then tape one end of the string to the centre of the base line of the protractor and tie the bolt to the other end of the string. I will check that the string hangs directly over the 90° mark when the protractor is held horizontally.



B and C. I chose the sign on the wall of my school because I can measure its height from the ground to check. The angle of elevation to the bottom of the sign is 11° when I am standing 2 m from the sign and holding the clinometer 1.5 m from the ground.
I drew a diagram and labelled it with the measurements I took to keep the information organized.



I used the tangent ratio of 11° to solve for x because I knew the length of the side adjacent to the 11° angle and side x is opposite the 11° angle.

$$\tan 11^\circ = \frac{x}{2}$$
$$2\tan 11^\circ = x$$
$$0.4 \doteq x$$
$$1.5 + 0.4 = 1.9$$

The sign is about 1.9 m above the ground.
I used a tape measure to measure the height of the bottom of the sign from the ground. It is 1.9 m from the ground, so my clinometer is accurate.

D. I will stand 10 m away from the wall of my school so i can see the roof clearly.

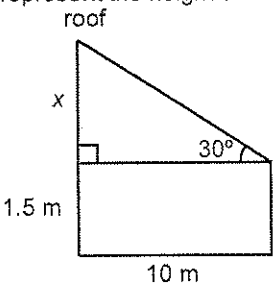
E. I will use the tangent ratio of the angle of elevation because I know its adjacent side (the distance from where I'm standing to the school, 10 m) and I need to know its opposite side (the height of the school above less 1.5 m, x). Some of my classmates are using the sine ratio, maybe because it is the first ratio in the list.

F. How a clinometer works:
Clinometers can be used, along with trigonometry, to determine the heights of tall objects that are hard to measure directly.

- You hold the clinometer with the straight edge of the protractor on top so that the string hangs down vertically over the protractor.
- You raise the clinometer to your eye, sight along the straw, and then angle the clinometer until you see the top of the building. Have someone else read and record the angle made by the string on the protractor at that point.

Once you have the angle of elevation to the top of an object, you can use trigonometry to determine the height. Here is what I did to determine the height of my school.

- I stood 10 m away from the school and used my clinometer (holding it at about 1.5 m above the ground) to measure the angle of elevation to the roof to be 30°.
- I drew a diagram and labelled it with the measurements I took to keep the information organized. I used x to represent the height of the school.



- I used the tangent ratio to solve for x.

$$\tan 30^\circ = \frac{x}{10}$$
$$10\tan 30^\circ = x$$
$$5.8 \text{ m} \doteq x$$

- I added the 1.5 m height of the clinometer to x to determine the height of the school.

$1.5 + 5.8 = 7.3$
My school is about 7.3 m tall.